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CHARACTERISTICS OF USAGE OF SUPPLY ITEMS ABOARD NAVAL SHIPS AND THE SIGNIFICANCE TO SUPPLY MANAGEMENT

Commander John Henry Garrett, Jr., SC, USN

EDITOR'S NOTE: This paper presents naval problems in supply of materiel. Research is being conducted at the George Washington University Logistics Research Project in connection with effecting the highest degree of self-sufficiency of fleets at sea. The Bureau of Supplies and Accounts and the George Washington University Logistics Research Project are jointly conducting a research program known as "The Allowance List Test Program." One of the notable features of this program is that military worth (essentiality) measurements for repair parts are being introduced explicitly.

INTRODUCTION

An analysis of Fleet demands for material in the Navy Supply System reveals highly significant characteristics. An examination of the implications of such characteristics upon supply management highlights an area of serious organizational weakness in the physical distribution of this material. Failure to take corrective action can lead to a further adverse effect upon Fleet readiness.

In order to arrive at the proper solution for this shortcoming, this paper examines organizational policies of successful businesses, both large and small. It concentrates, however, upon the larger, more complex ones, as these are the companies that begin to have management problems of some relationship to those of the Navy Supply System and its means for physical distribution.

It becomes increasingly clear when studying these companies that an overriding organizational principle, extending to the function of physical distribution, is the real key to successful management of the more complex companies. It is most apropos that the lessons of these companies be diligently studied and applied. Application of such lessons, and the principle the lessons so clearly reveal, can lead to a successful solution of the Navy's supply management problem of physical distribution to ships at sea in distant areas.

At times I may appear to be overly critical of some Department of Defense actions. In no sense should this be looked upon as disloyalty to my superiors whom I am sworn to uphold. This paper is simply written with complete intellectual freedom of thought and expression. It has attempted, as Professors Masland and Radway urge in their recent book Soldiers and Scholars, to challenge "accepted military thought or doctrine" and to "give- - -attention to critical analytical thought."[1] Differences of opinion on controversial issues are thus

[1] Masland, John W. and Radway, Lawrence, I., Soldiers and Scholars, Princeton University Press, Princeton, 1957.

unescapable. Otherwise any such paper as this could only be a namby-pamby apery of an existing situation. Such is not my purpose.

DEMAND PATTERNS

Even the most casual observer of the Navy Supply System is immediately impressed with the wide spread in the pattern of demand of Navy stocks.

Just to orientate our thinking to common ground, let's take a look at a few significant characteristics of this demand pattern.

Up to the present time most formal data collection programs which will show the cut of this pattern have been concerned with general stores material. However, sketchy information points quite clearly towards similar patterns in demand of technical stores.[2] First though, the story of material under the inventory management of the General Stores Supply Office (GSSO).

The AKS[3] load list[4] of general stores material of January 1955 had a range of 6800 different stock numbers out of a total of around 60,000 items in general stores system stock.[5] The issue effectiveness (ratio of supply to demand) of the AKS was around 80%.[6] Of special significance is the pattern of movement of the 6800 items, revealed by the following statistics of AKS issues of general stores material in the Atlantic and Pacific during the period 1 November 1954 - 30 April 1955:[7]

Number of times a particular stock number was issued	Quantity of stock numbers represented	% of the total range of the load list
PACIFIC OPERATIONAL AREA:		
100 to 825 (highest)	135	2%
50 to 99	249	4%
10 to 49	1929	28%
2 to 9	2786	41%
1 time	696	10%
Not issued	1010	15%
	6805	100%
ATLANTIC OPERATIONAL AREA:		
10 to 81 (highest)	147	2%
2 to 9	793	12%
1 time	2171	32%
Not issued	3692	54%
	6803	100%

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[2] Strictly speaking, however, general stores material has become largely technical material in recent years. The General Stores Supply Office has determined in its work on the coordinated shipboard allowance list program that about 95% of the general stores material in allowance lists is in direct support of specific equipments (CINCLANTFLT ltr ser 2706/440 of 18 June 1957, enclosure (8), p. 7).

[3] Fleet Stores Issue Ship.

[4] A specified list of stores, by stock number and quantity, for loading aboard a fleet issue ship. The list is prepared on the basis of supply items "most likely" to be required by a fleet, taking into consideration past usage data, equipments in the fleet, population, fleet deployment and engineering judgment. These lists make no pretence of supplying all of the requirements, only the "best sellers."

[5] Minutes of ComServLant Conference of 29 November 1955 on Load List Review.

[6] ComServLant ltr ser 80/13656 of 8 December 1955.

[7] Ibid.

In both fleets 2% of the items on this load list, which is equivalent to 0.2% of all general stores material in the system, represents the bulk of fleet issues. This reveals a sharp concentration of issue frequency in a relatively few stock numbers of general stores material.

Looking into the supply records of several ships to determine the internal final usage of general stores material, the Fleet Supply Research teams of Commander, Service Force, U. S. Atlantic Fleet uncovered an interesting situation along similar lines. These teams studied shipboard general stores requisitions and stock record cards covering a three year period. During this period the following situation on three ships was discovered:[8]

SHIP	Number of general stores items on board or requisitioned during period of 3 years	Number of items on board with no usage	Number of items stocked in excess of three years supply
USS SHASTA (AE-6)	3,868	1,766	365
USS ELOKOMIN (AO-55)	4,034	714	299
USS AMPHION (AR-13)	10,528	2,369	1,645

Here is more evidence that aboard individual ships, usage of general stores material is restricted to a relatively few items. But fleet-wide the General Stores Supply Office must carry 60,000 items in its system to meet all demands.

Next let's take a look at electronic repair parts—a highly significant category of repair parts today. Of the 200,000 such items cataloged at the Electronic Supply Office in May 1956, only 0.54% of them accounted for 50% of the total issues in the system. 22.08% accounted for 95% of the total issues, leaving 77% of the total range of stock to supply the remaining 5% of the demand. 27,000 electronic repair parts (17% of the number of items in the system) in the AKS load list meets around 85% of the Sixth Fleet demands. Of these, only 56 items were issued six or more times during the first calendar quarter of 1955.[9]

Statistics with the same categorical breakdowns are not available to me for other classes of repair parts. However, what statistics are available reveal a similar pattern.

While the destroyer tender, the USS SHENANDOAH (AD-26), was on station with the Sixth Fleet from 27 October 1954 to 21 February 1955, 1700 demands for ordnance repair parts were received. 81% were fulfilled. 76% of the demands were for stock numbers requested only one time.[10] 8,448 items (5.3% of the items in the Ordnance Repair Parts System) were on the AKS load list at the same time, which provided an average effectiveness (ratio of issues to demands) of 65%.[11]

Ship repair parts demands aboard the SHENANDOAH developed likewise. 660 demands were received and 92% of the items were supplied.[12] Again, a relatively high percentage, 64%, were demands of a one time nature.

Now to a broader base. Of the 183,369 items of ship repair parts in the supply system as of 31 Dec 1955, 11,465 items (6.2%) in the AKS load list provide an average effectiveness

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[8]ComServLant ltr ser 80/8015 of 31 August 1956.
[9]Nisbet, H. S., Cdr., SC, USN, ESO System Planning Division, in personal ltr dtd 23 May 1956.
[10]USS SHENANDOAH (AD-26) ltr ser 595 of 28 Mar 1955.
[11]Prosch, E. J., System Planning Division, Ordnance Supply Office, in personal ltr dtd 7 Dec 1956.
[12]USS SHENANDOAH ltr op. cit.

(ratio of issues to demands) of 70%.[13] On a still broader base, the 92,386 items (50% of the number of items in the system) of ship repair parts at the Naval Supply Center, Norfolk, on 31 December 1955 accounted for only 16.4% of the Ship Parts Supply System issues for the quarter ending on the date.[14] Similarly, 90,077 line items of ship repair parts (representing 49% of the number in the system) at NSC Oakland during the period accounted for only 18% of the system issues.[15] At the inland supply depots an even more static stock-turn situation prevailed during this same period. NSD Mechanicsburg had 130,860 items (71% of the number of line items in the system). These accounted for only 5.3% of the system issues. At NSD, Clearfield, 108,591 items (59%) accounted for only 3.3% of the system issues for the period.[16]

Probably the most detailed study to date of technical repair parts demands in the Navy was undertaken jointly by the Advanced Supply System Research and Development Division of the Bureau of Supplies and Accounts and the Logistics Research Project of George Washington University. This study analyzed 4 years of usage data of submarine repair parts and ship repair parts for 12 submarines. This usage covered ship's use, tender use and overhaul and alterations. The study revealed that the number of times an item was demanded during the four year period was extremely low. During this period, the number of times an item was demanded only once ranged from 64% to 78% for the twelve submarines.

The investigation could not determine how many submarine repair parts were installed on any of the sample submarines. However, if the number was similar to a count they were able to obtain of another submarine, 74% of the installed submarine repair parts would not have required replacement even once on the average sample submarine.

A similar demand pattern was revealed for the ship repair parts used on the 12 sample submarines.

It was concluded that "usage of [submarine repair parts and ship repair parts] on individual submarines are so low and so sporadic as to exclude the applicability of conventional inventory control techniques for shipboard stocking problems." A further conclusion was that more emphasis is required on the problem of range of items for an individual submarine's support than upon depth of stocking.[17]

WHAT IS THE SIGNIFICANCE?

What is the significance to supply management of this mass of statistics on the issue of supply system stock? What are the implications of the large percentage of issues being concentrated in a few items, while the entire range of demand necessary to support of the Fleet is scattered in an everchanging pattern composed of tens of thousands of items?

ENDURANCE LOADING

For one, they verify a long standing judgment of many persons with knowledge of the Navy Supply System that only a small percentage of items in the supply system account for a

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[13] Everett, Robert T., Cdr., SC, USN, Ships Parts Control Center, in personal ltr dated 5 June 1956.

[14] SPCC Internal Notice 4440 of 12 April 1956.

[15] Ibid.

[16] Ibid.

[17] Denicoff, Marvin; Fennel, Joseph P., and Solomon, Henry, "Requirements Determination," Progress Report No. 1, serial 59/57 of 4 June 1957. Advanced Supply System Research and Development Division, Bureau of Supplies and Accounts, and Logistics Research Project, George Washington University. Contract N7onr41904, Project NR 047001.

high percentage of Fleet demands. It was on the basis of such findings that endurance loading[18] evolved as a matter of Navy policy.[19]

DECREASING SELF-SUFFICIENCY OF SHIPS

Of even greater significance than the substantiation of the endurance loading concept, these statistics reveal the exceedingly broad range of repair parts necessary to keep our Navy operating. While a relatively few items of stock represent the bulk of the Fleet demand, and can be isolated into satisfactory load lists, there are so many items in the whole range of Navy stocks that it is simply infeasible to load the full range of potential demands about fleet issues ships, or to include them all in ships' allowance lists. But, still those items that have this fleeting kaleidoscopic demand pattern are just as important to Fleet readiness as the 75%-85% of the demands that can be met from the ship's allowance of repair parts or by fleet issues ships at sea with the Fleet in distant areas.

This heavy dependence upon support outside the ship has not always been the case, however. Our thinking may consequently be a little overly colored by our past experience with the mechanical skills of Navy crews, and with the great American talent of improvising. While these are still valuable logistic tools, they simply do not meet the requirements of today's technology to the degree they formerly did in the day's of far simpler equipments.

To illustrate an implication of the growing technology, visualize the problem of a burnt out radio coil twenty years age. A radioman worth his salt could rewind it from standard wire carried aboard any Navy ship. Today however, repairing some of the components of shipboard equipment requires the skills and facilities found only at the factory. New replacement components must be available to the ship. As the usage statistics reveal, the range of these repair parts requirements is exceedingly broad, and growing broader as more and more complex equipments come into operational use. As one example, in 1937 only 60 electronic tubes were required aboard a Navy destroyer. In 1952 the number had increased to 2209.[20] A look into

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[18]Garrett, J. H., "A Scheme for Improvement in Fleet Support," Journal of the American Society of Naval Engineers, Inc., August 1956, p. 527-532.

This concept envisions loading a combat ship with stores of relatively small cube requirements, taking into consideration the size of the individual item and its usage, in sufficient quantity to last from overhaul to overhaul.

The first ship slated for an endurance load of general stores material was the USS SHASTA, AE-6 (ComServLant ltr ser 80/8015 of 21 Aug 1956). January 3, 1956 was the date of the commencement of the detailed study of the usage date aboard the SHASTA, which led to the realization of its endurance load of general stores material on 30 October 1956 (Minutes of ComServLant Force Supply Officer's Weekly Meeting of 30 Oct 1956). This was accomplished under the direction of the first Atlantic Fleet Supply Research Team, headed by Chief Pay Clerk John J. Michura, USN, working under the Atlantic Fleet Supply Officer, RADM Hugh C. Haynsworth, Jr., SC, USN.

The first idea along these lines was proposed in early 1954 by Captain David F. Kinert, USN, at that time on the Staff of CINCLANTFLT. He suggested that the slow moving, bulky items aboard ship be loaded on the basis of 12 months issues. Minutes of the Atlantic Fleet Supply Officer's Conferences from the time of this proposal until early 1955 reveal that this germ of an idea was expanded to that of the present concept. The endurance loading concept was formally presented to the Chief of Naval Operations by ComServLant in September 1955 (ComServLant ltr ser 80/10557 of 7 Sept 1955).

The term, "endurance loading," for the overhaul to overhaul loading concept, was suggested at the Atlantic Fleet Force Supply Officers' Conference held at the General Stores Supply Office on 6 July 1955. It seems appropriate at this time that formal recognition be made of the originator of a term that has now become so commonplace in the Navy. He was Chief Pay Clerk T. H. Carlson, USN, then of the Technical Branch of the General Stores Supply Office.

[19]OpNav Inst. 4442.1A of 20 Dec 1956.

[20]NavSandA Publication 256, December 1953.

the allowance list of one of our newest destroyers reveals over 4000 installed tubes in its equipment. Over a 6000% increase in 20 years!

Thus we see a big change from the days when ships of the line could sail so equipped and supplied that they could operate completely self-sufficient for extended cruises. Occasional stops for water, provisions, and infrequent Naval stores were the essential contacts with logistic sources.

A reasonably ready source of supply of repair parts has thus become as necessary to a man-of-war's might as a ready source of fuel and ammunition.

EXPANDING TECHNOLOGY HAS A GREAT IMPACT ON LOGISTICS

This growing dependence upon technology has made this ready supply of technical repair parts a primary supply management problem. It must be tackled with boldness and intelligence. Although these parts are mostly small in size,[21] and generally low in cost,[22] they are very large in importance. Their delivery to requiring ships at sea in an expeditious manner has become one of the utmost importance.

The importance of this is illustrated by a statement of Admiral Arleigh A. Burke, USN before the Committee on Armed Services, House of Representatives on 18 January 1956, in which he said, "Our striking power is proportionate to our ability to replenish at sea in distant areas."[23] I am sure that Admiral Burke was not excluding those items of supply, so unpredicable in demand that they cannot be included in fleet issue load lists or in ship's individual allowance lists.

It is only natural that the airplane should be expected to play a major role in transporting the high priority fleet freight to the fleet "at sea in distant areas." Consequently, means for rapid delivery of essential requirements has been recognized as a vital part of naval logistics and is incorporated in the Navy's air logistic concept.[24] Unfortunately, however, this cannot be accomplished by a simple shrug of the shoulders and a wave of a hand after the decision, "we'll fly it to them."

EXPERIENCES WITH MILITARY TRANSPORTATION

While the general situation in military transportation is well known, an examination of some experiences of naval commanders with it should be made.

Commander Fleet Air Western Pacific has stated that "many factors combine to make Fleet readiness acutely dependent on rapid, responsive and adequate air transportation—not only transportation across the Pacific from Continental U. S., but throughout the Western Pacific Area."[25] Among these factors are the shrinkage of personnel in both numbers and qualifications, placing special emphasis on rapid travel of contractor's technical representatives and Navy specialists in critical functional fields in order to keep highly complicated electronic equipment, machinery and aircraft in an acceptable state of material readiness. Also, because of an acute shortage of certain repair parts, this command found it necessary to pool critical spares in centralized locations, depending upon rapid delivery to requiring

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[21] Bureau of Supplies and Accounts (Planning Division), "The Modern Air Logistic Concept, Project 'FAST' - Fleet Air Support Test," p. 6.

[22] As an example, 93% of the electronic repair parts on allowance lists cost less than $3.00 each. (CINCLANTFLT ltr ser 2706/440 of 18 June 1957, enclosure (8) p. 10.)

[23] OpNav Notice 5720 of 26 Jan 1956.

[24] Assistant Secretary of the Navy for Air memorandum serial 067 of 26 October 1955.

[25] COMFAIRWESTPAC Staff Study, "Air Logistic Support in the Western Pacific," July 1956.

activities wherever they may be (afloat and shore) in the Western Pacific. After citing numerous instances of poor support and cumbersome and time consuming administrative and communications procedures, this commander reached the conclusion that the overly centralized control of airlift (both Military Air Transportation Service and Fleet Logistic Air Wing) served to effect adversely his operational readiness, due to the lack of sufficient responsiveness to his requirements. Much of this lack of responsiveness seemed to hinge largely upon a philosophy of the statistical achievement of "low operating costs per ton mile" rather than a spirit of a "fire engine wise" readiness. Commander Fleet Air Western Pacific further considered that trunk routes in the Western Pacific strongly tended to be partial to the "volume" customers, thus requiring a time consuming shuttle service (by both air and surface) from large terminal facilities to naval activities, who are the relatively "small" volume customers.[26]

Another study of air logistics for an earlier period in the Western Pacific revealed similar problems. In his Korean War evaluation, Commander in Chief, Pacific Fleet stated that "it was not unusual for high priority cargo to take longer getting from its initial unloading point at Haneda, or Atsugi [MATS terminals in Japan] to ultimate destination, than it took, for the material to reach Japan from the Continental U. S. This was particularly true for small [naval] activities—or for ships [at sea] . . . poor roads and slow surface transportation [for movement of priority air freight!] further slowed down the movement."[27]

When Commander Air Early Warning Wing, Pacific Fleet implemented the aviation phase of the Pacific early warning network, he concluded that the necessary responsiveness in airlift could only be achieved through decentralized control. In order to provide a means for meeting his continuing emergency requirements he felt impelled to request assignment of a transport aircraft to his command for this purpose.[28]

The Fleet Air Support Test, Atlantic, (FASTLANT) conducted from 1 October 1956 to 31 March 1957, highlighted similar problems of slowness of delivery and lack of responsiveness in an air transportation system under high level centralized control. As a result of this test CINCLANTFLT concluded that while the test was successful from the standpoint of improving logistic readiness of the Sixth Fleet, the test highlighted areas requiring improvement in the Navy Supply System operations, including its air distribution system.[29]

It would not tax the imagination of anyone familiar with distribution to visualize the time lags, paper work and other complicating problems that would arise from the following typical movement and handling of FASTLANT priority emergency cargo:[30]

1. From NSC, Norfolk, via air and truck to Charleston, S. C. Air Force Base. (Other continental supply points frequently shipped to Charleston, S. C., by Navy commercial air charger, QUICKTRANS.) (Average time - 1-1/2 days.)
2. Offloading and assembly in the Air Force Terminal at Charleston Air Force Base.
3. Transfer to open ramp storage on the day air shipment was made.
4. Plane loading at Charleston AFB. (Average time - 1-1/2 days for steps 2, 3 and 4.)
5. Offloading to open storage at the Air Force Base at Nouasseur, Morocco to await transfer to the Naval Air Station at Port Lyautey, Morocco.

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[26] Ibid p. 34.
[27] CINCPACFLT Korean War, Evaluation Report No. 6, Interim 1 Feb 53 - 27 July 53, Chapter 11, p. 11-86.
[28] Commander Air Early Warning Wing, Pacific Staff, Study, "Implementation of the Aviation Phase, Pacific Barrier," 31 May 1956.
[29] CINCLANTFLT ltr ser 1891/440 of 3 May 1957
[30] Ibid.

6. Loading on trucks for hauling over the road from Nouasseur to Port Lyautey (approximately 100 miles). (An apparent repetition of COMFAIRWESTPAC's concern about trunk routes favoring "volume" customers.) (Average time — 1 day for steps 5 and 6.)

7. Offloading to open storage to await air lift to Naples or some other point.

8. Plane loading at Port Lyautey and offloading at Naples or some other point. (Average time for steps 7 and 8 - 4-1/2 days.)

9. Loading and air lift by COD flights to aircraft carriers or loading on opportune shipping in port. (Average time - 1-1/2 days.)

10. Highline or helicopter transfer from aircraft carrier or other ship to units of the fleet with the possibility of more than one transfer before reaching the requisitioning ship.

The average time for the entire transaction—from the date the priority emergency requisition was submitted until its receipt by the requesting ship—was 16-1/2 days. The average time for priority routine material was 44 days.

As may be expected serious delays occurred from time to time in moving cargo from Nouasseur to Lyautey. Diversion of air lift at Port Lyautey to meet other requirements (which can always happen when a commander does not have control) resulted at one time in such a backlog at Port Lyautey that a LST had to lift a major portion of the backlogged high priority air cargo.

The lack of an air freight terminal at sea with the facilities and organization to make further delivery to requiring ships also showed up as a handicap in FASTLANT. The use of combat aircraft carriers for this purpose has been repeatedly found to be unsatisfactory on other recent occasions.[31] Lack of satisfactory facilities and organization for the final leg in the distribution system also plagued the Commander of the Seventh Fleet during the Korean War. As a result CINCPACFLT, when noting the successful technique of carrier onboard delivery service (COD) that "was confined to carriers and, to a lesser degree ships operating with carriers," recommended that the COD service be continued and that "better air lift be provided to dispersed afloat units [other than carriers]."

A most serious problem that developed in FASTLANT was the periodic lack of MATS air lift from the U. S. Once all departures between Charleston and Nouasseur were cancelled for 12 days, shorter interruptions also being frequent. Unexpected substitution by MATS of small aircraft from time to time occasionally added further to the delay of the movement of large cube shipments.

The system of allocations to the Navy by the truly high-echelon Joint Military Transportation Committee (JMTC) brought more problems. Changes in air lift allocations, made on short notice prior to and during an allocation period, seriously reduced the air lift available. These changes were sometimes only a week or so apart. Although during the test the allocations were generally equal to the requirements submitted, a widely fluctuating relationship existed before the test commenced and were found to reoccur immediately after the test ceased. For example the May 1957 requirement submitted was 190 tons, the allocation was 134 tons.[32]

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[31] In particular, CINCLANTFLT ltr 836/440 of 4 March 1957 and CNO AdHoc Committee "Study of Sea and Air Transportation System" (Weakley Report) 3 Oct - 30 Nov 1956.

[32] CINCLANTFLT ltr ser 1891/440 Op. Cit.

CENTRALIZED CONTROLLED SURFACE LIFT ALSO UNRESPONSIVE

Dependence upon a surface transportation system under high level centralized control to deliver material to the "small" fleet customer seems to have similar short-comings. During the Korean War, CINCPACFLT found that "the scattering of resupply cargo in any bottom available commits irrevocably the mobile support ship to the port where the cargo is consigned and destroys most of the advantages, inherent in a mobile support force. The replenishment cargo required to sustain a general issue ship should be carried in resupply bottoms that have the same flexibility and responsiveness to a logistic commander's orders as the issue ship."[33]

COMSERVLANT has encountered the same problem with a centralized surface transportation system: "We are continuing to experience difficulties in arranging special lifts through MSTSLANT. It is felt that greater flexibility is needed in order to meet fleet requirements which call for points of discharge at other than those used by commercial ship lines."[34]

COMPLAINTS OF HIGH LEVEL COMMANDERS HAVE SAME PATTERN

At this point it is clear that numerous naval commanders with great and grave responsibilities have common complaints about the transportation system available to them for delivery of material essential to the fulfillment of their assigned tasks. It should be noted also that these commanders are principally concerned about delivery to forces constantly on the move, that are seldom near the large terminal facilities served by trunk line carriers. Essentially, their repeated complaints can be boiled down to the lack of adequate responsiveness to deliver their requirements, when they are needed, where they are needed.

Our sister service, the Air Force, appears to have similar complaints. The Rand Corporation, a major Air Force research agency, has made studies of the demands for aircraft parts. They too have found that many parts are used so infrequently that their demand is simply unpredictable. They have similarly concluded that a vital need exists for a logistic system that can provide these little used parts promptly when required by the combat squadrons.[35]

THE CULT OF UNIFORMITY

Our military transportation system seems to have become engrossed in the growing "cult of uniformity" of our society where more concern exists for uniformity than for adaptation of actions to clearly "ununiform" situations. Certainly some adherence to a minimum set of rules is desirable in both society and gainful operations. If everyone were rugged individualists, life would be a state of chaos. But there are decided limits to uniformity where departures from normal should be clearly recognized and accepted. Specialized and responsive transportation for the movement of high priority material to mobile forces "at sea in distant areas" clearly seems to be one of the necessary departures. I think that sufficient lessons of history have been documented in this paper to show quite adequately that the present operations of service transportation is simply not suitable for the needs of the Navy.

Transportation is not the only victim of this cult of uniformity, however. Some other examples of the extent to which centralization of operations, and its resulting attempts at uniformity, have gone should be known to get the proper "feel" of the atmosphere in which the military transportation system operates.

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[33] CINCPACFLT Korean War, Op. Cit. p. 11-22.
[34] COMSERVLANT Minutes of Weekly Force Supply Officer's Conference of 26 February 1957.
[35] Ferguson, Allen R., "Air Force Logistics," Aeronautical Engineering Review, January 1957.

Snowballing out of this regimented cult is a growing list of examples of bureaucratic procedures that curtail responsiveness to local conditions. Unless the cyclical process is checked, it can serve to build bigger and bigger top echelon organization where the problem of digestion of information for decision making can only become slower and slower.

The following examples of Department of Defense directives illustrate this striving for uniformity, relegating the local commander's judgement of the local situation to a smaller and smaller aspect of command. (The underlining of words in the following tabulation has been done to show emphasis on this point.)

1. "Procurement, Replacement and Utilization of Filing Cabinets,[36] for the purpose "to establish procedures and standards" governing this aspect of filing cabinets within the Department of Defense.

2. "Refuse Collection and Disposal"[37] for the purpose "to establish standards" (including frequency of service) for refuse collection and disposal.

3. "Use of Treated Wood in Buildings and Structures Maintenance"[38] for the purpose of "establishing standards" for the use of such wood in maintenance, repair, and rehabilitation of buildings or other structures, and the preservative of wood products in storage at Department of Defense Installations.

4. "Standards and Criteria for Construction - Requirements Criteria, Design Criteria and Space Allowance for Parking for Nonorganization Vehicles,"[39] for the purpose "to establish basic guides."

5. "Loading Rules, Test Loadings and Test Shipments Governing Rail Shipments"[40] for the purpose of providing "uniform requirements and procedures with respect to" the subject indicated in the title.

6. "Adjustment of Fluoride Content of Communal Water Supplies at Military Installations."[41] This cites DOD attitude concerning adjustment of fluoride content in communal water supplies at military installations and promulgates "general procedural guide lines" to implement this attitude.

7. "Standards for Drinking Water"[42] which "establishes DOD objectives and policies relating to the quality of water used for drinking and culinary purposes by the military departments."

A real down to earth example of the direct results of this kind of centralized operational philosophy is illustrated by the problem of getting a commissary store for 7000 eligible personnel on Taiwan. It took a four star admiral over 6 months simply to get administrative approval to use certain funds just to acquire a building for the store.

While these may be exceptional examples, they do indicate some of the limits to which centralizing of authority can be carried and some of the general atmosphere controlling the movement of vitally needed repair parts to its ultimate destination aboard men-of-war "at sea in distant areas."

They also indicate the far more serious general problem of the continuing loss of the commander's control over his logistics by overly controlled high level administrative action. But that is a story in itself and not the primary purpose of this paper.

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[36]DOD Instruction 4140.8 of 2 Nov 1955.
[37]DOD Instruction 4150.8 of 20 Oct 1955.
[38]DOD Instruction 4165.19 of 1 Aug 1955.
[39]DOD Instruction 4270.13 of June 30, 1955.
[40]DOD Instruction 4500.12 of Sept 1, 1955.
[41]DOD Instruction 6230.2 of Aug 30, 1956.
[42]DOD Instruction 6230.1 of Aug 18, 1955.

IMPROPER APPLICATION OF A GOOD CONCEPT

While the air logistic concept has been well conceived in broad terms, in my opinion, its application has been improper, largely because of organizational shortcomings which ignore certain vital and fundamental principles. Before such a concept will work, it is essential that these principles be recognized and applied. Otherwise in the minds of many partially informed people, the concept will be a failure. Whereas the facts will be that it failed because of poor organizational application, rather than of a conceptual weakness.

OBJECTIVES OF A TRANSPORTATION SYSTEM

There are three objectives that one can choose for an air transportation system, or for that matter any transportation system.

1. It can be operated for maximum efficiency and effectiveness from the standpoint of carrier operations, i.e., maximize lift per dollar cost and per hour of operations. An example is the economic advantage to a carrier of awaiting movement of cargo until a full load has been accumulated.

2. It can be operated for maximum efficiency and effectiveness of a supply distribution system, i.e., a short (in time) pipeline, resulting in lower system stock levels and decreased warehousing costs. An example here is the U. S. Air Force LOGAIR system, which has resulted in lower stock levels of certain aeronautical items, notably aircraft engines.[43] Civilian applications of this objective likewise appear to have merit,[44] and it is recognized as one objective of the Navy's air logistic concept.[45]

3. It can be operated for maximum economy and effectiveness of the consumer. In the case of the military, the consumer is the combat commander at the scene of action, who has the immediately applicable information required to make sound decisions. Such an objective in a military transportation system would put high price on immediately responsive delivery service. A long standing Navy example is the space expensive technique of combat loading amphibious shipping, so that material can be unloaded and placed ashore in the troops' hands expediously after the combat commander, (who alone has the necessary applicable information at hand), makes the decision that such material is required to further the accomplishment of his mission. Such an objective thus would serve to promote a fire-engine type readiness transportation system.

While none of these objectives are sufficiently compatible for incorporation in a single transportation system under continuous centralized control, there is a recognized place in the military establishment for all three. We seem though, to have really only recognized the first two in actual practice, if we believe the testimony of numerous commanders.

The third objective of providing for maximum economy and effectiveness of the consumer has been largely buried under by the statistical advantage of the first two. Unfortunately for this last objective, statistics for the first two are more readily charted and understood. Economy and effectiveness of the consumer are hard to measure in understandable terms that can be balanced against the clear-cut statistics that can be mustered for the first two. This especially is true in the military service. Thus the measure of military air transportation is consistently that of the dollar type statistics of either higher tons per dollar or lower inventory

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[43] The Wall Street Journal. April 11, 1956, p. 1.
[44] Lewis, H. T., Culliton, J. W., and Steele, Jack D., The Role of Air Freight in Physical Distribution. Harvard University, Boston 1956.
[45] Assistant Secretary of the Navy for Air Memorandum, Op. Cit.

costs per dollar of transportation operations, all measured at the highest administrative level. I have never seen a like statistical measurement of military effectiveness, nor do I expect to see one in such clear terms. Yet no one can deny that military effectiveness is a changeable phenomenon, and that it is directly affected by the ready availability of needed supplies.

I believe that here we can well take the word of caution made by a highly successful industrialist, General Electric's President Ralph J. Cordiner. He has stated,* "To adopt a system or approach that relies primarily on statistics could easily foster a dangerous tendency to assume that anything which cannot be expressed numerically does not actually exist. Such a highly theoretical approach could lead to excessive rigidity, and to overlooking many of the most important factors. . ."[46]

After this word of caution, it is well at this point to turn to the business world and study the philosophy that guides their physical distribution procedures.

POLICIES UNDER WHICH PHYSICAL DISTRIBUTION IS MADE IN THE BUSINESS WORLD

An American business must, if it is to survive in the competitive system, produce profits. Its degree of success is measured directly in terms of degree of profits. Finding the key management technique employed in the successful companies should lead to some organizational principles that may well be applicable to the military problem of physical distribution.

A company that has experienced rather phenomenal growth in recent years is the Air Reduction Company, Incorporated, a manufacturer and distributor of industrial gases. The following extract† from a recent annual report is significant:[47]

"The company [corporation] is constantly examining its organization in the light of expanding and diversifying markets. The Air Reduction Sales Company is the largest single operating division of the Company. In view of its recent substantial growth, a thorough study of its organization was made in 1956. As a consequence, three integrated marketing territories were established, as of January 1, 1957. Each is headed by a regional vice president of the Division who is responsible for production, distribution, sales and accounting (underlining supplied). The divisional staff organization at headquarters was also strengthened and duties more clearly defined.

"The decisions based on the study were announced by the president of Air Reduction Sales Company Division in Mid-October, so that ample time was allowed for each member of the organization to plan for his new responsibilities. The net effect is further to decentralize responsibility and place the company in a stronger competitive position." (Underlining supplied.)

A comparison of the number of employees at the end of 1946 and the end of 1956 reveals interesting facts. During this 10 year period the number of employees increased about 3-1/2 percent, the number of salaried employees decreased 3 percent. During this same period the company's sales rose 138% and earnings per share of common stock 260%. The long term debt decreased and the property account increased 260%! Exploiting management brains, judgement and initiative at a low level management and giving him the logistic tools to accomplish his job obviously paid off here.

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*Reprinted by permission from NEW FRONTIERS FOR PROFESSIONAL MANAGERS by Ralph J. Cordiner. Copyright, 1956. McGraw-Hill Book Company, Inc.

†Permission to reprint the above has been granted by the Air Reduction Company.

[46]Cordiner, Ralph J., New Frontiers for Professional Managers copyright, 1956. McGraw-Hill Book Co., New York. Reprinted by permission of McGraw-Hill.

[47]Air Reduction Co. 41st Annual Report - 1956.

Another example of success in the business world bears examination. Ralph J. Cordiner, General Electric Company head, makes an exceedingly strong case for decentralization of management in his company.[48] The company's sales have risen from 200 million dollars in 1920 to 3 billion in 1955. Mr. Cordiner, a highly respected industrial leader, recognized when he joined the company that such growth characteristics required an entirely different managerial approach in 1950 than in 1920. His conclusion was that the company required "increasingly better planning, greater flexibility and faster, more informed decisions than was possible under the highly centralized organizational structure [existing when he became head]."[49] His solution was a philosophy of management decentralization, not according to products, geography of function types of work, but rather decentralization of responsibility and authority for making business decisions.[50] Thus "the individual managerial and functional employees, who have the immediately applicable information required to make sound decisions and take prompt action [have the responsibility, with commensurate authority and accountability for making business decisions]."*[51]

Against the background of the Department of Defense's tendency toward centralized authority in logistic areas, General Electric's ten guiding principles are of more than routine interest:[52]

"1. Decentralization places authority to make decisions at points as near as possible to where actions take place.

"2. Decentralization is likely to get best over-all results by getting greatest and most directly applicable knowledge and most timely understanding actually into play on the greatest number of decisions.

"3. Decentralization will work if real authority is delegated; and not if details then have to be reported, or, worse yet, if they have to be "checked" first.

"4. Decentralization requires confidence that associates in decentralized positions will have the capacity to make sound decisions in the majority of cases; and such confidence starts at the executive level. Unless the President and all the other officers have a deep personal conviction and an active desire to decentralize full decision-making responsibility and authority, actual decentralization will never take place. The Officers must set an example in the art of full delegation.

"5. Decentralization requires understanding that the main role of staff or services is the rendering of assistance and advice to line operators through a relatively few experienced people, so that those making decisions can themselves make them correctly.

"6. Decentralization requires realization that the natural aggregate of many individually sound decisions will be better for the business and for the public than centrally planned and controlled decisions.

"7. Decentralization rests on the need to have general business objectives, organization structure, relationships, policies, and measurements known, understood, and followed; but realizing that definition of policies does not necessarily mean uniformity of methods of executing such policies in decentralized operations.

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[48] Cordiner, Ralph J., Op. Cit., reprinted by permission of McGraw-Hill.

[49] Ibid, p. 45, reprinted by permission of McGraw-Hill.

[50] Ibid, p. 47, reprinted by permission of McGraw-Hill.

[51] Ibid, p. 49, reprinted by permission of McGraw-Hill.

[52] Ibid, p. 50-52, reprinted by permission of McGraw-Hill.

"8. Decentralization can be achieved only when higher executives realize that authority genuinely delegated to lower echelons cannot, in fact, also be retained by them. We have, today, Officers and Managers who still believe in decentralization down to themselves and no further. By paying lip-service to decentralization, but actually reviewing detailed work and decisions and continually "second-guessing" their associates, such Officers keep their organization in confusion and prevent the growth of self-reliant men.

"9. Decentralization will work only if responsibility commensurate with decision-making authority is truly accepted and exercised at all levels.

"10. Decentralization requires personnel policies based on measured performance, enforced standards, rewards for good performance, and removal for incapacity or poor performance."*

Still another characteristic of the General Electric decentralization philosophy is of particular interest, especially to those with knowledge of the intense effort made to avoid duplication and promote standardization in the military departments. Two of General Electric's divisions, Hotpoint Division and Major Appliance and Television Receiver Division directly compete with each other with such appliances as refrigerators, ranges and home laundry equipment. They have different facilities, different product designs, different distribution and different prices. "They compete at the market place very aggressively, and, incidently, very profitable."*[53]

A look into another segment of industry reveals still another example of increased profits through the philosophy of decentralization.[54] When President Robert A. Magowan took over the operation of Safeway Stores in late 1955, of all the major food chains Safeway had the doubtful distinction of being low man on the totem pole of profits. 1956 was the biggest and best year Safeway has ever had—sales were up 3% and profits up 87%.[55]

Development of local autonomy through decentralization accounted largely for this turnabout. The merchandizing function was turned over completely to the 18 distribution division managers. Nobody from the "home office" tells them how to sell groceries. Nobody tells them how to procure (which includes the physical distribution of groceries), nor for that matter even gives them any guidance on which brands to buy. The division managers are not even obligated to try to sell Safeway brand names! They are only accountable for end results.

General Dynamics Corporation, which was a 14-million dollar business in 1946, crossed the billion dollar sales figure in 1956. Here, as with other dynamic companies, the watch word is decentralization.[56] Under President Frank Pace, Jr. the division managers have a major degree of autonomy and are given the credit by Mr. Pace for the success of his company. The "home office" staff is only 100 people for dealing with fiscal, legal and public relations matters. When policy decisions are to be made on production, sales and research they are handled by the management board, which includes the division managers. This method of operation fits in with former Secretary of the Army Pace's often expressed policy of decentralization. And profits and sales continue to rise.

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*Reprinted by permission from NEW FRONTIERS FOR PROFESSIONAL MANAGERS by Ralph J. Cordiner. Copyright, 1956. McGraw-Hill Book Company, Inc.

[53] Ibid, p. 59, reprinted by permission of McGraw-Hill.

[54] Business Week, April 13, 1957, p. 63-74.

[55] Results for 1957 look even better, in the first six months sales were up 6.7% and profits were up 40% over the first 6 months of 1956. (New York Times, July 9, 1957.)

[56] Business Week, May 11, 1957, p. 61-66.

The profitable manner in which divisions of another industrial giant, General Motors, compete with each other in procurement, design, distribution and sales are too well known to need documentation in this paper.

A study of the role of air transportation in physical distribution in the business world has been the subject of a recent study conducted at Harvard University. The study team found that the decision to use air freight is almost always made by "top management."[57] The top management in the industries studied by this team bears no resemblance, however, to top management in the military establishment. The Harvard University report discusses only relatively small businesses, such as watch manufacturing plants, electron tube producers, printing corporations and the like. None of their "top managements" even approach the management responsibilities of a fleet commander, for example, much less the Naval or Defense Establishment.

DOES THE SUCCESSFUL BUSINESS PRINCIPLE OF DECENTRALIZATION HAVE A PLACE IN THE MILITARY SERVICES?

While this look into the business world may seem to have strayed from the main point—that of distribution of non-load list technical repair parts to the fleet units "at sea in distant areas"—the profitable philosophy of decentralization in business appears to have a definite place in the non profitable business of warfare. As has been noted in the foregoing discussion, in the business world the decision making point on such matters as distribution are made at a managerial level low enough to have the "immediately applicable information required to make sound decisions and take prompt action." The business world manager is also given the authority, responsibility and tools to carry out his decision, the soundness of this decision being judged by one of life's strictest disciplines—produce satisfactory profits in a competitive capitalistic system or be sacked.

It seems a perfectly direct analogy to me to look upon a task fleet commander as the very highest point at which similar decisions should be made regarding the distribution of the technical repair parts essential to maintaining his competitive position—his military readiness. We must recognize that for comparative purposes, military readiness and its "profits" are not adaptable to measurement by the statistician as are the dollars in a business ledger. No one can deny the real unmeasurable value of constant military readiness, nor the enormous cost in today's world if the lack of military readiness is exploited by a "competitor."

RESPONSIVENESS IN MILITARY TRANSPORTATION AND DOLLAR ECONOMY ARE NOT COMPATIBLE

It is a serious mistake to expect all phases of military transportation, particularly that part which should be set aside with the objective of providing maximum economy and effectiveness to the consumer, to operate with high utilization, high payloads and carry only commodities which justify air transportation on economic grounds. The permanent diversion of aircraft to this objective must be looked upon as a peak-load problem, such as is found in many industries.[58] The question must be centered upon the needs of the combat commander rather than a blind judging of military transportation by commercial standards.

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[57] Lewis, Culliton, and Steele, Op. Cit. p. 54.

[58] For example, railroads have periodic excesses of boxcars, steel plants have periodic excesses of steel making capacity, canners have excess canning capacity in off seasons, and farmers have excess plows in the winter. Nevertheless they find that profits generated during their full utilization during peak-load periods more than offsets the non profitable idle periods.

I think that it has been shown that all is not rosy in the military transportation service, particularly air. The experiences of recent history also show that much of the trouble can be traced to the inflexibility resulting from the centralization of control in the military transportation system.

In view of this the December 13, 1956 Department of Defense order[59] directing even greater centralization of military air transportation is most disturbing. It was reported the new directive "appeared to be a crackdown to end interservice duplication [of airlift]."[60] Presumably it is expected that statistics on cost per ton mile will show a lowering of dollar cost. (I hope that this can be balanced in some statistical manner as to its effect upon fleet operations!)

RECENT INDICATIONS OF A DEPARTMENT OF DEFENSE CHANGE OF PHILOSOPHY OF OPERATIONS

An encouraging short range trend has been recently noted, however, former Secretary of Defense Wilson once made some very pointed remarks opposing any more unification, tion, stating that "the alternative [to the then current Pentagon organization] would be the over-concentration of power and authority in the hands of one or two men. —This would risk a form of dictatorship in which we do not believe."[61]

At a press conference on 16 May 1957, the former Secretary of Defense indicated a further reversal of a pattern observed by Mr. Eugene S. Duffield: "All the debates, orders and laws since 1947 thus fall into a single pattern: Starting from the 1947 concept of decentralization along horizontal lines of coordination, the trend has run steadily and swiftly toward centralization on a single vertical line of command."[62] On May 16, 1957 Mr. Wilson said:

"If we concentrated all our purchasing and our estimates of requirements in one place you'd have a whole lot more paper work than we have now. You can't possibly concentrate all the purchasing of common use items in one supply. It's too big. You wouldn't dare risk it for the security of the country. You would make it more difficult for small business to have any part in it, and only by decentralizing can you handle this difficult one of not only what you pay for things, but the quality you get in the item and whether you get it in proper timing with when you need it, and I assure you that is done better by decentralization than it is by centralization. (Underlining supplied.)

"My former experience, about every three years someone would raise the question wouldn't we save money in General Motors if we centralized the thing and did all the buying in one big place, pointing out that the Buick Company paid a few cents more a thousand for some nuts or bolts or screws of something, all the common items, and we would look it over again and decide no, that it wasn't the right way to do it, and it was much more efficient and effective, and our big problem was to decentralize the thing and clarify the policy, simplify the administration and promote efficiency and avoid the concentration of stupidity, and that is what I have been working on down here, too, the same way."[63]

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[59] New York Times, December 14, 1956, p. 1.
[60] Ibid.
[61] New York Times, June 12, 1957, p. 7.
[62] Duffield, Eugene, S., "Organization for Defense," Harvard Business Review, Sept-Oct. 1953, p. 38.
[63] Department of Defense news release of Secretary of Defense C. E. Wilson's press conference of May 16, 1957.

Organizationally speaking, Mr. Wilson made it clear that the philosophy of organization that is good for General Motors is good for the Defense Department. Such sentiments certainly are in keeping with the repeated recommendations on organization for physical distribution made by responsible naval commanders.

Application of this philosophy of decentralization in the organization for the distribution of high priority material for fleets operating in distant areas is well in order.

SUMMARY AND CONCLUSIONS

In summary, certain highlights stand out:

1. While a certain segment of U. S. Navy supply items has definite usage patterns so that demands for these items can be reasonably predicted, there is a very large range of items with no usage pattern and whose demand is simply unpredictable.

2. Those with reasonably predictable demands lend themselves to the endurance loading concept, fleet issue load lists, and individual ship's allowance lists.

3. Those with unpredictable demands are so vast in number that it is simply infeasible to include them all in ship's allowances or on fleet issue load lists.

4. Those with unpredictable demands are often as essential to proper fleet readiness as those whose demands can be reasonably predicted and are included either in ships' allowances or in fleet issue load lists. They are consequently needed in the shortest possible time after the demand is generated.

5. Growing technology is expanding the range of unpredictable demands for repair parts and at the same time making their ready availability even more essential to Fleet readiness.

6. A distribution system for delivery of items not included in either allowance or load list to naval ships "at sea in distant areas" is essential for Fleet readiness.

7. The prevailing philosophy of centralization in organization in the Department of Defense precludes the essential degree of responsiveness in the transportation aspects of the Navy distribution system to supply the items of unpredictable demand within a satisfactory time period.

8. The highly profitable philosophy of decentralization in the business world appears to have direct application in certain aspects of transportation in support of the fleets operating in distant areas.

9. There appears to be a decided requirement for specialized air terminal facilities at sea in the near vicinity of fleet units.

Three developments are highly desirable, if not essential, for the insurance of an acceptable degree of Fleet readiness.

First, there must be available to the Navy a highly responsive air transportation system, with a "fire-engine" ready point of view. This can only be achieved under an organizational philosophy of decentralization with an objective of maximum contribution to the efficiency and effectiveness of the combat commander. Such a system must never be judged on any basis of cost per ton mile, nor cost per transport operational hour, nor cost of supply inventory. The sole criteria for judging should be the degree of required service given to the fleet commander in delivering the high priority freight required to preserve the readiness of his ships.

Secondly these should be included in the mobile logistic support forces, logistic support ships, with a flight deck and other facilities necessary to make it an efficient air freight terminal and with the capability of delivering air freight directly to the requiring ship.[64]

Thirdly, sufficient logistic aircraft of a design compatible with such a transportation system and terminal facilities, must be made available to the fleet commander operating in distant areas.

Compromises with either of these three developments, such as superimposing these tasks upon existing systems or facilities, can only result in less than a satisfactory outcome. The Fleet customer is just too valuable and too necessary to the safety of our country to be jeopardized by continuation of the inadequate support with which he must now struggle.

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[64] Details of such a ship are discussed in a previous article, "A Scheme for Improvement in Fleet Support," Journal of the American Society of Naval Engineers, Inc., August 1956 and in the Weakley Report, "Study of Sea and Air Transportation Systems," 3 October--30 November 1956, Op. Cit.

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A BOTTLENECK SITUATION INVOLVING INTERDEPENDENT INDUSTRIES*

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In this paper, we consider the computational solution of a maximization problem arising in the study of the utilization of interdependent industries. Under the assumption of proportional costs and returns, it is shown that the dimensionality of the problem can always be reduced by one and all the transformations occurring can be taken to be "shrinking transformations." These transformations greatly improve the efficiency of the method.

INTRODUCTION

In earlier papers, [1, 2], using the functional-equation technique of dynamic programming, we discussed a mathematical model associated with the economic question of the most efficient utilization of a complex of interdependent industries. In this paper, we discuss the numerical solution of a particular class of problems of this general type. As we shall show, a simple device affords a great reduction in computation time.

A "BOTTLENECK" SITUATION

Let us assume that we have two interdependent industries, the "auto" industry and the "steel" industry, and that the state of each industry at any particular time may be completely specified by two quantities, the stockpile of raw material required for production and the maximum productive capacity. To simplify the problem of this initial computation, we shall assume that the auto capacity is unbounded. The purpose of the process will be taken to be the production of as many autos as possible in a time interval of T_0 periods.

The process is assumed to be discrete, with allocations made only at time $t = 0, 1, 2, \ldots, T - 1$. At any particular one of these times t, let

$$x_s(t) = \text{amount of steel in the steel stockpile,}$$
$$x_m(t) = \text{capacity of steel mills.} \tag{1}$$

At each time t, the steel in the stockpile may be allocated in any proportion to the following three purposes: to produce additional steel by using the existing steel capacity, to increase the steel capacity, or to produce autos by using the existing auto capacity.

*Manuscript received March 1958.
†This paper originally appeared as RAND paper P-1282 of April 17, 1957.

Let us write

$$x_s(t) = z_s(t) + z_m(t) + z_a(t), \tag{2}$$

where

$z_s(t)$ = the quantity of steel used to produce additional steel, (3)

$z_m(t)$ = the quantity of steel used to increase steel capacity,

$z_a(t)$ = the quantity of steel used to produce autos.

Let us impose the following constraints:

$$z_a(t) \leq k_1 x_s(t), \quad 0 < k_1 < 1, \tag{4}$$

$$z_s(t) \leq x_m(t).$$

The first constraint says that it is not possible to use more than a fixed percentage of steel for auto production over any stage, t to t + 1, while the second asserts that there is no point to allocating more steel to the steel mills than the maximum capacity.

Let us now see how the state of the system is affected by these allocations. If we assume linearity of production,

$$x_s(t+1) = k_2 z_s(t), \quad k_2 > 1, \tag{5}$$

$$x_m(t+1) = x_m(t) + k_3 z_m(t), \quad k_3 > 0.$$

Finally, let us assume that units are so chosen that the quantity of autos produced in a stage is $z_a(t)$.

It is required to choose the quantities $z_s(t)$, $z_a(t)$, and $z_m(t)$, for $t = 0, 1, 2, \ldots, T_0 - 1$, so as to maximize the total quantity of autos produced over the time period $[0, T_0]$, from the initial quantities,

$$c_1 = x_s(0), \tag{6}$$

$$c_2 = x_m(0).$$

LINEAR-PROGRAMMING APPROACH

The problem reduces to that of maximizing the linear form

$$L(z) = \sum_{t=0}^{T_0 - 1} z_a(t), \tag{7}$$

subject to the constraints of Eqs. (2), (4), and (5), a problem within the domain of linear programming.

Let us, however, count variables, assuming that we are interested in a 30-step process. Taking three unknowns at each stage, we have a problem involving 90 variables, subject to 120 relations. Although this problem is not formidable in terms of the simplex method, it is sizable. If we wish to determine the dependence of the solution on the initial parameters c_1 and c_2, this method is unwieldy, since it requires a separate computation for each pair of values c_1 and c_2. In its place, we wish to present a method that yields the solution for all positive c_1 and c_2 by means of the computation of a sequence of one-dimensional functions.

DYNAMIC-PROGRAMMING APPROACH

Let us define, for $T = 1, 2, \ldots,$

(8) $f_T(c_1, c_2)$ = total auto production over T steps, when initial quantities c_1 and c_2 are used at start and an optimal policy is employed.

Clearly, we have

$$f_1(c_1, c_2) = k_1 c_1 . \tag{9}$$

Using the principle of optimality, we obtain

$$f_R(c_1, c_2) = \operatorname*{Max}_{\{z\}} [z_a(0) + f_{R-1}(k_2 z_s(0), c_2 + k_3 z_m(0))] , \tag{10}$$

for $R = 2, 3, \ldots, T$, where the maximization is over the region in z space defined by

(11) (a) $z_a(0), z_s(0), z_m(0) \geq 0$,

(b) $z_a(0) + z_s(0) + z_m(0) = c_1$,

(c) $z_a(0) \leq k_1 c_1 = k_1 x_s(0)$,

(d) $z_s(0) \leq c_2 = x_m(0)$.

Henceforth, we shall let z_a, z_s, and z_m denote $z_a(0)$, $z_s(0)$, and $z_m(0)$, the allocations for the first stage of the process.

In the next section we shall discuss the numerical determination of the sequence $\{f_R(c_1, c_2)\}$ for $R = 1, 2, \ldots,$ and $c_1, c_2 \geq 0$.

REDUCTION TO A SEARCH OF VERTICES

The region in the (z_a, z_s, z_m) plane determined by the inequalities (Eq. (11)), has the form shown in Figure 1.

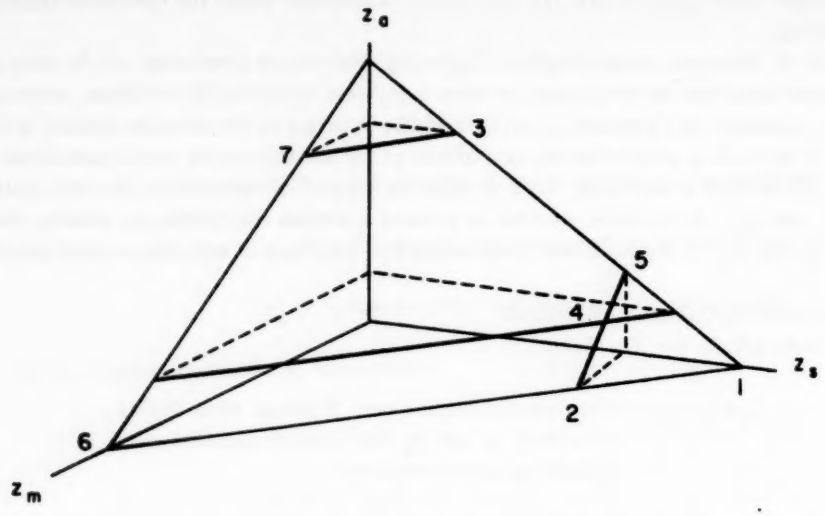

Figure 1 - Region of variability of the quantities z_a, z_s, z_m

The vertices, as numbered in Figure 1, have the following significance in terms of the process:

1. In case mill capacity does not represent a constraint, all available steel is allocated to the production of more steel.
2. As much steel as possible is allocated to the production of steel; when the mill capacity is met, the rest is allocated to the expansion of mill capacity.
3. The allowable percentage of steel stockpile is assigned to auto production, the rest to steel production.
4. This vertex occurs when mill capacity is insufficient to handle all the steel available. It represents an allocation of steel to steel production up to the mill capacity, an allocation to auto production up to the allowable percentage, and an allocation of the remaining steel to the expansion of mill capacity.
5. As much steel as allowable is allocated to steel production and the rest to auto production.
6. All of the current steel stockpile is allocated to the expansion of mill capacity.
7. As many autos as possible are produced and then mill capacity is expanded with the remaining steel.

Obviously all of these conditions do not occur at once, but depend on steel capacity and steel stockpile, which vary throughout the process.

It may be shown, either from the functional equation, Eq. (10), or as a consequence of known results in the theory of linear inequalities, that the maximization over the region defined by Eq. (11) reduces to a maximization over the seven possible vertices marked in Figure 1.

Of these seven, only the first five are actual possibilities, since vertices 6 and 7 effectively end the process—no new steel being produced.

The stagewise maximization problem is thus quite trivial. The tabulation problem, however, remains nontrivial because of the possibility of an expanding (c_1, c_2) domain as R increases. This difficulty will be overcome in the following section.

REDUCTION IN DIMENSION AND MAGNITUDE

Let us now show that we can reduce the problem to a sequence of one-dimensional problems and simultaneously introduce shrinking transformations.

It is first of all clear from the linearity of all the constraints and production functions that $f_T(c_1, c_2)$ is a homogeneous function of c_1 and c_2 of the first degree. Hence, for $c_1, c_2 > 0$, we have

$$f_T(c_1, c_2) = c_1 f_T\left(1, \frac{c_2}{c_1}\right) \tag{12}$$

$$= c_2 f_T\left(\frac{c_1}{c_2}, 1\right)$$

It follows that we need compute only $f_T(1, x)$ or $f_T(x, 1)$. Turning to Eq. (10), we have

$$f_R(1, c_2) = \underset{\{z\}}{\text{Max}} \left[z_a + f_{R-1}(k_2 z_s, c_2 + k_3 z_m) \right] \tag{13}$$

$$= \underset{\{z\}}{\text{Max}} \left[z_a + k_2 z_s \, f_{R-1}\left(1, \frac{c_2 + k_3 z_m}{k_2 z_s}\right) \right]$$

We see, then, that the calculation of $f_R(1, c_2)$ for $c_2 \geq 0$ depends only on a knowledge of $f_{R-1}(1, c_2)$ for $c_2 \geq 0$. This is the required reduction in dimensionality. However, we still face the difficulty of an expanding range for c_2.

In order to avoid this difficulty, let us show that we can compute $f_R(1, x)$ and $f_R(x, 1)$ for $0 \leq x \leq 1$, knowing $f_{R-1}(1, x)$ and $f_{R-1}(x, 1)$ for $0 \leq x \leq 1$.

Referring to Eq. (13), we have

$$f_R(1, c_2) = \underset{\{z\}}{\text{Max}} \left[z_a + k_2 z_s \, f_{R-1}\left(1, \frac{c_2 + k_3 z_m}{k_2 z_s}\right) \right] \tag{14}$$

$$\text{for } k_2 z_s \geq c_2 + k_3 z_m ,$$

$$= \underset{\{z\}}{\text{Max}} \left[z_a + (c_2 + k_3 z_m) f_{R-1}\left(\frac{k_2 z_s}{c_2 + k_3 z_m}, 1\right) \right]$$

$$\text{for } k_2 z_s \leq c_2 + k_3 z_m .$$

Combining this equation with the foregoing observation concerning the maximization over vertices, we have a fairly simple computational scheme.

COMPUTATIONAL TECHNIQUE

The simplifications introduced by the transformation technique of the previous section represent the most significant contribution of this particular study. With respect to the programming, the results are (a) a reduction in time and space requirements from T^2 to $2T$ and (b) a considerable further reduction afforded by the elimination of expanding grid. Let us elaborate on these two points.

Normally, to express all possible states of a system defined by two independent parameters (here c_1, steel stockpile, and c_2, mill capacity), it is necessary to construct a grid of $f_T(c_1, c_2)$ in (c_1, c_2) space and then to interpolate over this two-dimensional region to determine $f_T(c_1', c_2')$, the steel allocable to auto production during a T-period process in which the initial conditions are c_1' and c_2' and an optimal policy is pursued. This function is necessary for our recursive calculation of $f_{T+1}(c_1, c_2)$. If the intervals $[0, c_1]$ and $[0, c_2]$ are divided into n parts by this grid, we must compute and store n^2 values of $f_T(c_1, c_2)$ for future use. The time requirement becomes even more serious because of the extra logic needed when a two-dimensional system is dealt with. This accounts for the savings resulting from the reduction to one-dimensional form.

The possibility of an expanding grid is a serious obstacle in some dynamic-programming processes. Nonmathematically, the situation is this: To calculate the conditions at time t, we must know in advance all possible states that might exist at time t - 1. In this particular application, to determine auto production over T periods, we must know auto production for T - 1 periods for all allowable steel stockpiles and capacities. But after one period of production, either stockpile or capacity, or both, may be increased. So, to calculate $f_T(c_1, c_2)$, $f_{T-1}(c_1', c_2')$ is needed, where c_1' may be greater than c_1 and similarly for c_2'. Consequently, the region over which $f_T(c_1, c_2)$ can be calculated is smaller than the region over which $f_{T-1}(c_1, c_2)$ is known. One must therefore begin a T-stage calculation by considering a large region in order to complete the calculation with a modest range of values for c_1 and c_2. The technique of the previous section, by-passing this obstacle, represents a real and significant advance.

One further innovation in this problem, the optimization over a three-dimensional region, bears mention. Techniques for the solution of general problems of this multidimensional sort have been little investigated. Here, of course, we are saved by the nature of the functions and have shown that only the vertices of the region need be considered. The coding technique used in order to determine and evaluate the relevant vertices is diagrammed in Figure 2.

The remainder of the program is concerned with the calculation of a table of values $f_T(c_1, c_2)$, the block transfer of this table, and its subsequent use in the derivation of a table of $f_{T+1}(c_1, c_2)$.

Table 1 presents the result of an analysis of a typical situation. The calculation generates the optimal choice of vertex at each stage for each initial condition and also lists the total achievable steel allocation to auto production. To better display the results, a hand calculation was performed, by using the policy dictated by the computer; this shows the actual,

unnormalized growth of the system as a function of time. The sensitivity of the process was demonstrated by evaluating the return from a policy that was optimal in all but the first decision. An initial choice of vertex 3 resulted in an over-all reduction in productivity of 8 percent.

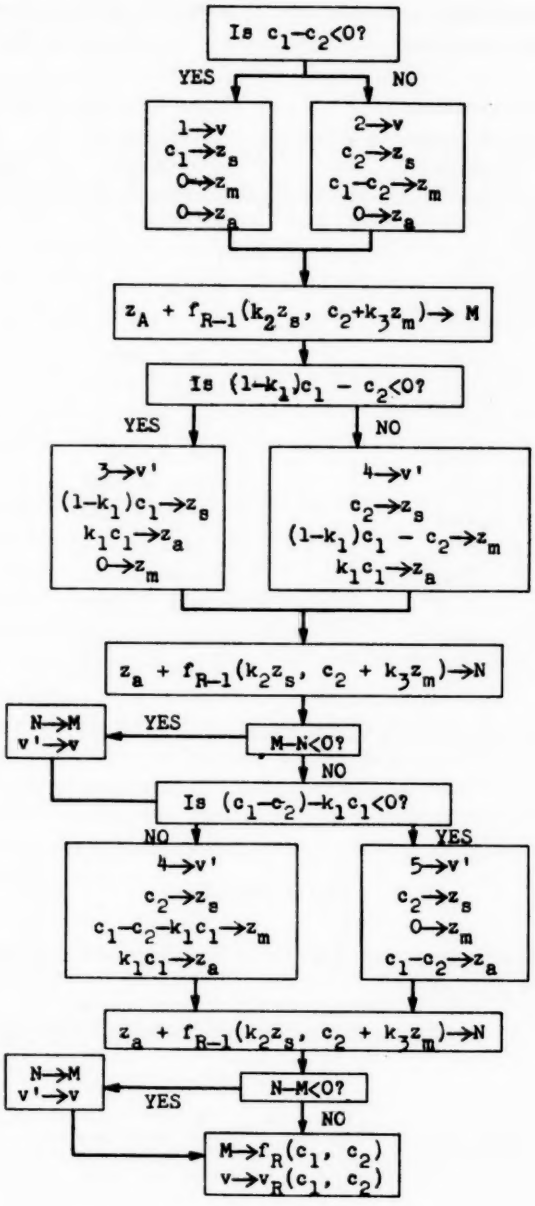
Is $c_1-c_2<0$?
YES
NO
$1\to v$ $c_1\to z_s$ $0\to z_m$ $0\to z_a$
$2\to v$ $c_2\to z_s$ $c_1-c_2\to z_m$ $0\to z_a$
$z_A + f_{R-1}(k_2 z_s,\ c_2+k_3 z_m)\to M$
Is $(1-k_1)c_1 - c_2<0$?
YES
NO
$3\to v'$ $(1-k_1)c_1\to z_s$ $k_1 c_1\to z_a$ $0\to z_m$
$4\to v'$ $c_2\to z_s$ $(1-k_1)c_1 - c_2\to z_m$ $k_1 c_1\to z_a$
$z_a + f_{R-1}(k_2 z_s,\ c_2 + k_3 z_m)\to N$
$N\to M$ $v'\to v$
YES
$M-N<0$?
NO
Is $(c_1-c_2)-k_1 c_1<0$?
NO
YES
$4\to v'$ $c_2\to z_s$ $c_1-c_2-k_1 c_1\to z_m$ $k_1 c_1\to z_a$
$5\to v'$ $c_2\to z_s$ $0\to z_m$ $c_1-c_2\to z_a$
$z_a + f_{R-1}(k_2 z_s,\ c_2 + k_3 z_m)\to N$
$N\to M$ $v'\to v$
YES
$N-M<0$?
NO
$M\to f_R(c_1,\ c_2)$ $v\to v_R(c_1,\ c_2)$

Figure 2 - Routine for the choice of vertex

TABLE 1

Analysis of a Typical Bottleneck Situation
(Fifteen-Stage Process, with Parameters $k_1 = 0.2$, $k_2 = 2$, and $k_3 = 0.4$, and Initial Quantities $c_1 = 1$, $c_2 = 1$)

Information Made Available by Calculation			Actual Conditions When Optimal Policy is Used	
Stage	Normalized Condition (c_1, c_2) at Beginning of Stage	Vertex	Actual Condition at Beginning of Stage	Steel Allocated to Auto Production
1	(1, 1)	1	(1, 1)	0
2	(1, 0.5)	2	(2, 1)	0
3	(1, 0.7)	2	(2, 1.4)	0
4	(1, 0.5857)	2	(2.8, 1.64)	0
5	(1, 0.642)	2	(3.28, 2.096)	0
6	(1, 0.611)	2	(4.192, 2.57)	0
7	(1, 0.626)	2	(5.14, 3.22)	0
8	(1, 0.620)	4	(6.44, 3.988)	1.288
9	(1, 0.588)	4	(7.976, 4.4536)	1.595
10	(1, 0.586)	4	(8.9072, 5.2196)	1.781
11	(1, 0.573)	4	(10.4392, 5.9817)	2.008
12	(1, 0.579)	4	(11.9634, 6.9268)	2.393
13	(1, 0.576)	4	(13.8536, 7.9797)	2.771
14	(1, 0.577)	4	(15.9594, 9.2086)	3.192
15	(1, 0.577)	4	(18.4172, 10.6226)	3.683
Total allocation to auto production				18.711

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MULTIPLE SERVERS WITH LIMITED WAITING SPACE*

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The functioning of a service facility, such as those typically represented by waiting-line-theoretical models, is often influenced by limitations on the number of customers who can be kept waiting at one time. In extreme cases, there is a flat limit: customers arriving when the line has grown to a certain length automatically turn away or are turned away. The purpose of the present note is to investigate, for the simplest Poisson-exponential multiserver systems with such a flat limit on waiting-line length, the proportion of arriving customers which will be lost.

Specifically, a relationship will be obtained connecting the proportion of customers lost to the number of servers, the number of waiting spaces, and the ratio of workload potentially imposed to full-utilization capacity of the system. Tables and graphs based upon this relationship also are given.

It should be noted that the model to be considered here is appropriate for at least two types of problems differing slightly from that described above. In one, the only change is that customers who "turn away or are turned away" are not lost, but merely subcontracted or otherwise served outside the normal performance of the system (for instance in overtime periods). For the purpose of the present note it makes no difference whether it is the last arrival who is lost (or subcontracted, or what have you) or perhaps some earlier arrival chosen according to a priority rule.

The other type of problem to which the model of this paper is applicable is that in which an always-loaded input mechanism is "blocked" whenever the waiting line reaches its limiting length (as when one machine must stop for lack of waiting space at the machine center into which its product is fed). In this case, the proportion of customers "lost" is to be interpreted as the proportion of potential customers which never arrives, or as the proportionate utilization of the input mechanism.

THE PROBLEM

A system of N servers receives customers in a Poisson-type time series, with mean interarrival time $1/\lambda$, and serves them on the usual pooled-waiting-line basis, with holding time exponentially distributed and having mean $1/\mu$. However:

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There is some nonnegative integer W such that whenever a customer arrives to find N + W customers already in the system, then one customer is "subcontracted," so that the "state of the system" remains N + W, without a customer's having been served.

The ensuing paragraphs will show how to calculate how large W must be in order to limit the long-term-average proportion of subcontracted customers to a predesignated quantity, i, and will exhibit the results of some hand computations in tables and graphs.

THEORY

By using methods identical in form with those appearing in Refs. [1] and [2], it is easy to show that if p_k is the steady-state probability that the system will contain exactly k customers, then

$$p_k = \begin{cases} \dfrac{N^k}{k!}\,\rho^k\,p_0 \quad , & 0 \le k \le N \\ \dfrac{N^N}{N!}\,\rho^k\,p_0 \quad , & N \le k \le N + W \end{cases},$$

where $\rho = \lambda/N\mu$ (which need not be less than unity) and p_0 is defined by the requirement that $\sum_{0}^{N+W} p_k = 1$.

Now define u as the long-term-average proportion of the servers working (average "utilization"). This means

$$u = \frac{1}{N}\left[\sum_{1}^{N} k\,p_k + \sum_{N+1}^{N+W} N\,p_k\right]$$

After substitution of the p_k from above, a simple expression can be obtained for 1 - u, but before this expression is stated it will be convenient to define

$$K(\rho, N) = \frac{N!}{(N\rho)^N} \sum_{k=0}^{N-1} \frac{N-k}{N} \frac{(N\rho)^k}{k!}$$

Then:

$$1 - u = \frac{(1-\rho)\,K}{K - \rho^{W+1}}, \quad \text{for } \rho \neq 1 .$$

For $\rho = 1$, the expression just given is degenerate, whence a special case is necessary, as follows:

$$1 - u = \frac{1}{W + 1 + \dfrac{N!}{N^N} \displaystyle\sum_{0}^{N-1} \frac{N^k}{k!}} , \quad \text{for } \rho = 1 .$$

This can be obtained either directly or by applying L'Hôpital's rule to the general case.

To avoid extra words about relatively uninteresting cases, it will now be convenient to assume that $\rho \leq 1$.

Let i be the long-term-average proportion of arriving customers whose arrival results in a customer's being "subcontracted." Since ρ is the average proportion of the servers which would be working if all customers were ultimately served, and u is the actual average proportion of servers working, it is obvious that

$$i = (\rho - u) / \rho .$$

After some manipulation of the expression for $1 - u$, it turns out that

$$\rho W = \frac{K(\rho, N)}{\dfrac{1 - \rho}{i} + \rho} , \quad \text{for } \rho < 1 ,$$

and

$$W = \frac{1}{i} - \frac{N!}{N^N} \sum_{0}^{N} \frac{N^k}{k!} , \quad \text{for } \rho = 1 .$$

These equations can, of course, be interpreted as determining the value of W necessary in order to insure that i will be at some predesignated level. By thinking of W in this way, it is sensible to write it as a function of i, ρ, and N: $W = W(i, \rho, N)$.

Setting $N = 1$, it is then another matter of straightforward manipulation to show that the following expressions are correct:

$$W(i, \rho, 1) = \frac{\log\left(\dfrac{1 - \rho}{i} + \rho\right)}{\log(1 / \rho)} - 1 , \quad \text{for } \rho < 1 .$$

$$W(i, 1, 1) = \frac{1}{i} - 2 .$$

After some slight further manipulation for general N, it is seen to be convenient to make the following definition:

$$D(\rho, N) = \frac{\log K(\rho, N)}{\log (1/\rho)} - 1 \ , \quad \text{for } \rho < 1 \ ,$$

$$D(1, N) = \frac{N!}{N^N} \sum_{0}^{N-1} \frac{N^k}{k!} - 1 \ .$$

This definition makes it possible to write the following:

$$W(i, \rho, N) = W(i, \rho, 1) - D(\rho, N) \ .$$

This formulation expresses $W(i, \rho, N)$, for arbitrary N, as the difference between the value for a similar one-server system and a quantity dependent upon ρ and N alone. This representation is especially interesting because W is relatively easy to calculate for $N = 1$ and because D is relatively small.

Specifically, it is not difficult to show that, for $0 < \rho \leq 1$ and $N > 1$, the following holds:

$$0 < D(\rho, N) < N - 1 \ .$$

Table 3, below, indicates that D assumes values substantially smaller than the indicated upper bound in many of the "interesting" cases where utilization is fairly high.

TABLES AND GRAPHS

The function $K(\rho, N)$ appears in a number of expressions relating to systems such as are discussed here, and it seems appropriate to include, parenthetically, some numerical values (Table 1). The writer feels that it might be worthwhile to investigate further the general properties of $K(\rho, N)$, but is putting the present note together without having done so, in view of the fact that this function enters only relatively insignificantly into the expressions for $W(i, \rho, N)$.

TABLE 1

$K = K(\rho, N)$ for Selected (ρ, N) Combinations. (These figures can be assumed accurate to within 0.02 or 1.0 percent, whichever is larger.)

ρ	N					
	1	2	3	4	6	10
0.98	1.02	1.03	1.04	1.05	1.06	1.08
0.95	1.05	1.08	1.10	1.12	1.15	1.21
0.9	1.11	1.17	1.22	1.27	1.35	1.49
0.8	1.25	1.41	1.54	1.68	2.01	2.41
0.6	1.67	2.23	2.82	3.47	5.06	9.82
0.3	3.33	7.22	14.3	26.0	89.3	855

The principal computational results presented herewith are contained in Tables 2 and 3, the first of which tabulates $W(i, \rho, 1)$ and the second of which tabulates $D(\rho, N)$. These are of slide-rule accuracy only. Figures 1 and 2 are visual representations of some of the data in or obtainable directly from Tables 2 and 3. Note that the horizontal scales of these graphs are logarithmic.

TABLE 2

$W(i,\rho,1) = W(i,\rho,N) + D(\rho,N)$.

Blanks indicate negative functional values (which will appear in a few additional squares when D is subtracted in computing W for $N > 1$). The figures can be assumed accurate to within 0.1 or 1.0 percent of their magnitude, whichever is larger.

i	ρ												
	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.85	0.9	0.95	0.98	0.99	1.0
$\frac{\sqrt{10}}{10}$	-	-	-	0.1	0.2	0.4	0.6	0.7	0.9	1.0	1.1	1.1	1.2
$\frac{1}{10}$	0.3	0.6	1.0	1.5	2.0	2.7	3.6	4.3	5.1	6.3	7.2	7.6	8.0
$\frac{\sqrt{10}}{100}$	1.0	1.6	2.2	3.0	4.1	5.5	7.8	9.6	12.3	17.0	22.7	25.6	29.6
$\frac{1}{100}$	1.7	2.5	3.5	4.7	6.3	8.6	12.6	16.0	21.6	33.8	53.1	67.6	98.0
$\frac{\sqrt{10}}{1000}$	2.4	3.5	4.7	6.3	8.5	11.8	17.7	22.9	31.9	53.8	97.5	141	314
$\frac{1}{1000}$	3.2	4.4	6.0	8.0	10.7	15.0	22.8	30.0	42.8	75.7	150	238	998
$\frac{\sqrt{10}}{10000}$	3.9	5.4	7.2	9.6	13.0	18.2	27.9	37.0	53.6	97.8	205	347	3160
$\frac{1}{10000}$	4.6	6.3	8.5	11.3	15.2	21.5	33.1	44.1	64.6	120	262	459	10000
$\frac{\sqrt{10}}{100000}$	5.3	7.3	9.7	13.0	17.5	24.7	38.2	51.2	75.5	142	319	572	31600
$\frac{1}{100000}$	6.0	8.2	11.0	14.6	19.7	27.9	43.3	58.3	86.5	165	376	687	100000

TABLE 3*

$D(\rho, N)$

N	ρ						
	0.3	0.6	0.8	0.9	0.95	0.98	1.0
2	0.7	0.6	0.5	0.5	0.5	0.5	0.5
3	1.2	1.0	0.9	0.9	0.9	0.9	0.9
4	1.7	1.5	1.3	1.3	1.3	1.2	1.2
6	2.8	2.3	2.1	1.8	1.8	1.8	1.8
10	4.6	3.7	2.9	2.8	2.8	2.8	2.7

*This table (especially with interpolation) permits correction of Table 2 figures to suit cases with $N > 1$, through the expression

$$W(i, \rho, N) = W(i, \rho, 1) - D(\rho, N).$$

It also permits one to determine how far the curves in Figs. 1 and 2 should be "lowered" in order to fit cases with $N > 1$. Figures accurate to 0.1.

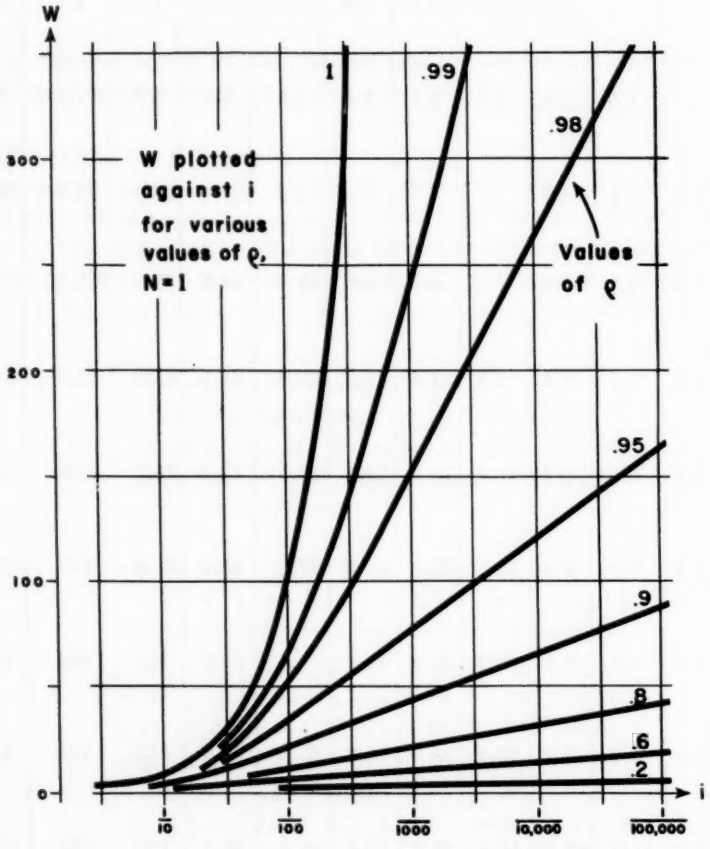

Figure 1 - W plotted against i for various ρ-values ($N = 1$)

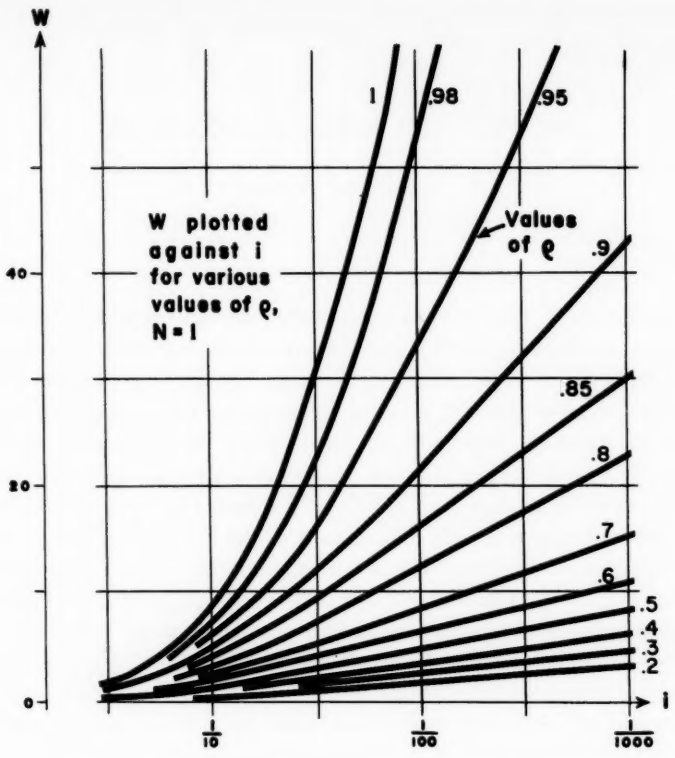

Figure 2 - Detail from Figure 1, w plotted against i for various ρ-values (N = 1)

It should be noted that the graphs can be read three ways. By reference to Figure 1, for instance, the following purposes can be served:

With allowable i, required ρ, the necessary W can be determined.
With available W, imposed ρ, the consequent i can be determined.
With available W, allowable i, the permissible ρ can be determined.

A much larger number of interpretations can be made if effects of variation in N are considered. The analytic expressions, of course, will serve these same purposes for arbitrary combinations of the variables concerned.

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OPERATIONS RESEARCH AND LOGISTICS

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One technical characteristic that differentiates the current work in operations and logistics research from other systematic quantitative methods of studying management problems, and usually one that is written in mathematical form, is the use of a parameterized model [15]. This is certainly not a technique new in the 20th century [27], but it is a methodology that is being used ever more widely and effectively as modern computational aids make it more feasible. This analytical technique has been greatly improved and extended since World War II with the help of such powerful new tools from applied mathematics as are provided by modern mathematical statistics and econometrics. It is in this modern trend that the topics for this paper have been selected, with full realization that the new that seems promising today may be either the common-place or the forgotten of tomorrow.

Included are a few logistics problems on which use may be made of various aspects of linear-programming theory. Mention is made of two other examples of the use of mathematical models in solving logistics problems: namely, in inventory control and preventive maintenance. Four special problems that could be included under linear programming theory are: budget planning, product distribution, personnel assignment, and production scheduling. The work reported was mostly done either by The RAND Corporation or under the Logistics Research Project at The Goerge Washington University, or by groups under contract with RAND or the Office of Naval Research.

Here is the main point. If one has a fairly precisely stated mathematical model, where the variables correspond to quantities relevant to a management problem, and if this model is in a field of mathematics that has been fairly well developed, then, of course, it is a primary virtue of our approach that we have at our disposal a large body of relevant mathematical theory. So, instead of each separate problem having to be thought through, it is very likely that some theorem in the literature, or some theorem easily proved by a mathematician, will take care of a large number of the more difficult logical steps otherwise involved in finding a solution.

Perhaps the simplest mathematical model available is a set of ordinary linear algebraic equations. This model has been chosen here not only because it is the simplest but also because it has rather wide application in industrial and military fields.

*An earlier version of this paper was written while the author was Professor of Industrial and Management Engineering and Director of the Institute for Research in the Management of Industrial Production at Columbia University. This earlier paper was presented at the First Ordnance Conference on Operations Research held at Frankford Arsenal during May 1954. The present paper is expository in character and is a revision of the earlier paper to bring it up to date.

$$a_{11}x_1 + a_{12}x_2 + \ldots a_{1n}x_n - b_1 = 0$$

(1) $$a_{21}x_1 + a_{22}x_2 + \ldots a_{2n}x_n - b_2 = 0$$

$$\ldots \text{ etc.}$$

$$a_{m1}x_1 + a_{m2}x_2 + \ldots a_{mn}x_n - b_m = 0$$

(1') $$Ax - b = 0 .$$

Formula (1) is simply a standardized way of writing down a system of linear equations where the x's are the variables. A primed formula, such as Formula (1'), is exactly the same as the unprimed one but often more convenient as an abbreviated notation.

Those readers not already familiar with such mathematical techniques may agree that the abbreviation seems to be a sensible one to use. It helps a great deal in the mathematics but is not particularly relevant for those who are not mathematicians. In (1') the little "x" is a vector, A is a matrix, and b is a vector. Similar statements hold between Formulas (2) and (2') or (3) and (3').

(2) $$x_1 \geq 0 , \; x_2 \geq 0 , \ldots \text{ etc.}, \; x_n \geq 0 .$$

(2') $$x \geq 0 .$$

(3) $$c_1x_1 + c_2x_2 + \ldots + c_nx_n = \text{minimum.}$$

(3') $$c' x = \min.$$

It is strictly true, from a mathematical standpoint, that any linear programming problem consists simply in solving a set of linear equations whose variables are nonnegative. It is this condition of nonnegativity that makes the field interesting. It makes it interesting and important because, in the practical applications, it is fundamentally for this reason that this particular model has rather wide application. The use only of linear restrictions may seem like a very special case, but in practice this is usually not so because, as is generally known, if enough variables are allowed one can fit almost any complicated function with a set of linear equations [21]. So it is a very general model indeed.

In passing, it is remarked that in the theory of games, or at least in the 2-person zero-sum case, one works with exactly the same type of linear restrictions on the variables [20]. So does one in several other currently popular mathematical models for operations and logistics applications.

In problems where one wants to take an economic point of view, it is sometimes desirable to add another condition—such as a "cost" one wishes to minimize. In Formula (3) one can think of the "c's" as being prices, and the "x's" as being quantities, so that the products of prices and quantities are added together to get cost. Since it is desirable to keep cost small, in various economic problems, Formula (3) is a very familiar kind of restriction.

Formula (4) is one of the standard forms for a linear programming problem.

$$Ax \geq b, \quad x \geq 0, \quad \text{and} \quad c'x = \min \text{ ("dual" of (5))}. \tag{4}$$

It can be written down in other forms, but this is a popular one. It is simply a set of linear inequalities that the nonnegative variables must satisfy and the economic condition that cost is to be minimum.

Formula (5) looks very much like Formula (4).

$$A'y \leq c, \quad y \geq 0, \quad \text{and} \quad b'y = \max \text{ ("dual" of (4))}. \tag{5}$$

Instead of a greater-than or equal sign, a less-than or equal sign is used, and profit maximization rather than cost minimization is implied. Although Formulas (4) and (5) look a bit different, it has been proved that if either problem has a solution so does the other; each problem is called the "dual" of the other. In other words, there is an x that will satisfy (4) if and only if there is a y satisfying (5); this "duality theorem" is an important one for these linear problems. Furthermore, if one has a problem in form (4) there is an equivalent problem that may be written as in Formula (6), which is called the "combined problem."

$$Ax \geq b, \quad A'y \leq c, \quad x \geq 0, \quad b'y = c'x. \tag{6}$$

A fundamental mathematical theorem states that problem (4) has a solution if and only if problem (6) has a solution [20]. Now notice that problem (6) no longer has an optimization condition—there is nothing apparently being minimized or maximized. This is the key step in these fields, in the sense that one can start the problem with an economic criterion and replace it simply by a set of restrictions on the variables, as in problem (6). This will be illustrated later for a few concrete logistics problems.

Models (7) and (7') are identical.

$$Ax \geq b, \quad -A'y \geq -c, \quad x \geq 0, \quad y \geq 0, \quad b'y - c'x \geq 0, \quad c'x - b'y \geq 0. \tag{7}$$

$$Ax - u = b, \quad A'y + v = c, \quad b'y - c'x - w = 0, \quad c'x - b'y - z = 0, \tag{7'}$$

$$x \geq 0, \quad y \geq 0, \quad u \geq 0, \quad v \geq 0, \quad w \geq 0, \quad z \geq 0.$$

This illustrates an important point, but one not stressed here especially, that it does not matter whether the model one uses is a system of linear equations in nonnegative variables or simply a general system of linear inequalities. One can transform back and forth between these two models, or between either and a system in which one includes a minimization condition on some linear combination of the variables. Formula (7) is simply a recasting of (6) so that the equalities have been eliminated; for example, if we want the condition that $b'y$ shall be equal to $c'x$, then one common way is to rewrite it as two inequalities requiring that the difference be both greater than or equal to zero and less than or equal to zero. Similarly, Formula (7') is written simply as a system of linear equations in nonnegative variables showing how to pass back and forth between systems of linear inequalities and systems of linear equalities. Here then are a few tricks of the trade used by mathematicians. So much, then, for a general background for the field of linear programming.

More interesting, perhaps, is the application of these notions to some practical problems of logistics. The first application discussed is to product distribution. Since there is now a considerable literature in this area, only one or two typical results are given here—with references to the literature for other recent work.

Formula (8) provides the model for the first and main example discussed, referred to as the Product Distribution Problem [13].

$$\sum_{i=1}^{m} x_{ij} = c_j , \quad \sum_{j=1}^{n} x_{ij} = r_i , \quad x_{ij} \geq 0 ,$$

(8)

$$\sum_{i=1}^{m} \sum_{j=1}^{n} x_{ij} d_{ij} = \min .$$

The notation was changed slightly, from that of Formula (4), for a variety of reasons. The x_{ij} are the now familiar nonnegative variables; it is convenient to label them with two subscripts instead of one, and there are m x n of these variables. The first two equalities in (8) are simply systems of linear equations where c_j, r_i and d_{ij} will turn out to be given constants that come from the distribution problem.

The first example discussed, and one easily translated into other problems more directly of interest to the reader, relates to the operation of the U. S. military tanker fleet in 1948-1949 [16]. Tankers go from Port i to Port j empty, at a cost of d_{ij}, and x_{ij} is the yearly number of them. The "slate" calls for c_i empty tankers to enter unloading Port i, and for r_j empty tankers to leave loading Port j, during the year. The last condition in (8) is the economic condition. Very similar conditions will appear in the production-scheduling examples discussed later.

What all this says is that the total cost of running the tanker fleet should be as small as possible, given only the number of full loads to be moved during the year from each loading Port j to each unloading Port i. Since the loaded-tanker schedule is completely specified by the slate, then the port handling costs also are completely determined, and this leaves only the pattern of movement for empty tankers to be decided—this is the only problem in the present example.

Figure 1 shows diagrammatically the shipping situation with which we are concerned. A tanker going empty from unloading Port 2 to loading Port 3 would go an "equivalent" distance d_{23} along the path $U_2 \rightarrow L_3$, where d_{23} really represents cost, including allowances for canal charges and the like. The variable x_{23}, similarly, represents the number of empty tankers travelling the route $U_2 \rightarrow L_3$ during the year. For example, if U_2 is Hamburg and L_3 is Bahrein then:

$d_{23} = 75$ means about 7500 miles,

$c_3 = 120$ means 120 full loads leave Bahrein or, alternatively, 120 empty tankers enter Bahrein, and

$r_2 = 141$ means 141 full loads enter Hamburg or, alternatively, 141 empty tankers leave Hamburg.

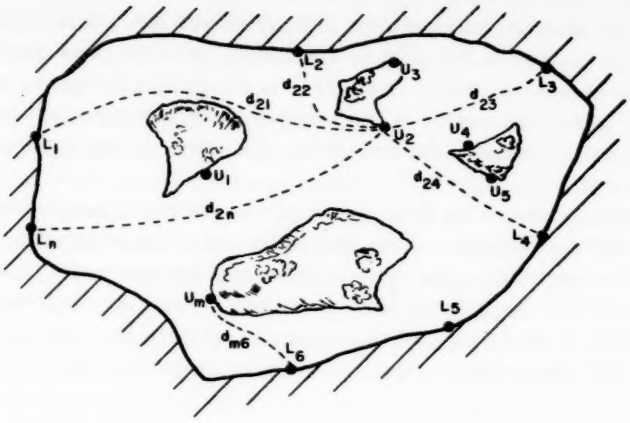

Figure 1 - Shipping diagram

In actuality, as this example was treated, there were 31 full loads to go from Bahrein to Hamburg, but the solution provided that only $x_{23} = 20$ empty tankers moved from Hamburg to Bahrein; the other empty tankers entering Bahrein came from Yokohama, Guam, Naples, and Manila.

Administratively, each Petroleum Operating Area estimates its total requirement at each unloading port for bulk petroleum products and the Armed Services Purchasing Agency then purchases these products for delivery at the specified loading ports. The Military Sea Transportation Service, operating the tanker fleet, then schedules the tankers so as to make the deliveries required by the POA slates and the ASPPA contracts.

There is an interesting mathematical problem involved in finding a set of values for the x_{ij} that will satisfy all the conditions of (8), when the numerical values are given for c_j, r_i, and d_{ij}. The combined problem that results from (8) is shown in (9).

$$\sum_i x_{ij} = c_j\,,\quad \sum_j x_{ij} = r_i\,,\quad d_{ij} + u_i - v_j = p_{ij} \geq 0\,,$$

(9)

$$x_{ij}\,p_{ij} = 0\,,\quad x_{ij} \geq 0\,.$$

In other words, (9) is to (8) as (6) is to (4). The economic condition is of course eliminated in (9), and it turns out that (9) is relatively easy to solve. We shall not consider here the mathematical and computational techniques used to solve for the x_{ij} in (8) or (9); suffice it to say that there are now routine programs for solving such problems, if neither m nor n is much greater than 100, that are practical on any of the larger, modern, digital computers.

Before considering the end result, there is an important point to be made. Obviously the shipping model is oversimplified. There is a great deal more to decide than just where to send tankers so as to get the smallest cost by some economic measure. There are "little" things that turn out to be important. For example, one cannot have a set of men who never get home for Christmas; nor can one safely assume that everything will go exactly according to schedule—it may turn out that somebody simply changed his mind during the year, or that war

started somewhere! Thus the solution one gets mathematically in this way may not really fit the case at hand. We need not point out other difficulties with this simple model; they are typical of every such problem in the author's experience. It is the same thing as with any other problem handled by an engineer; for example, if he is designing a bridge, he has to use a handbook on strength of materials, another book to answer design questions, and so forth. If he is a good engineer he will be aware of the limitations of his models, handbooks, and tables. That is equally true here.

We are not going far into the many assumptions back of this particular model for the tanker operation. We cannot, however, say this really solves the practical problem. Storage and handling facilities provide against many uncertainties that would otherwise need to be taken into account in the model. On the other hand, in hauling Av Gas there is considerable variation in monthly rate of use; if the storage facility is small, and there are fluctuations beyond those accommodated by the storage system, then just being accurate about the annual estimate is not good enough.

One of the first things the analyst always does in these problems is to start tacking on to the mathematical model such complicating factors as uncertainty of demand. The models rapidly get complicated in this particular instance, as was quickly found when account was taken of such factors as storage and other uncertainty aspects of the problem [14]. The central moral is that one can put in these additional factors provided one understands them precisely enough. One may even be able to put them into a simple linear system, but the number of variable may mount so rapidly as to get beyond manageability from a computational standpoint.

A concrete example of such difficulties is provided by one of the very earliest attempts to apply these general ideas to the operation of the Military Air Transport Service [1]. The number of equations and variables there was in the thousands. Even were the necessary data available, it would be completely impractical to consider computing with them. So the model was useless. The fundamental reason the approach was useless, and the model complex, was the frequency of transhipment. A load goes from here to there and is split up into several planeloads and reloaded, the reloaded plane going to several different next destinations. Taking direct account of this splitting up and reloading, throughout the system, made the number of equations and variables get tremendous. The model is not practical in that problem as of now. In the case of the military tanker fleet problem, it was simplified enough so that the number of variables was small and yet a realistic and useful answer could be computed from the model.

A computer could be used to handle various versions of the tanker-scheduling problem. The important thing gained by this would not necessarily be economy in carrying out the calculations but rather a shortening of the length of time it would take to recompute the operation. It is hard to stress this point too much. It is an important point about such models that: if one understands the operation well enough to describe it mathematically, and then wants quickly to make an operational change, use of a high-speed computer may indeed cost more in dollars yet make the calculation possible in time. Moreover, access to an adequate computer may be far cheaper in the long run than the cost of keeping a large number of trained clerical personnel. This seems to be one of the main advantages in using fancy computers on problems requiring intermittent but rapid solutions.

Turn next to the Assignment Problem. Formula (10) is a special case of Formula (8), but the interpretation is a little different.

$$(10) \qquad \sum_{i=1}^{m} x_{ij} = \sum_{j=1}^{n} x_{ij} = 1 , \quad \sum_{i,\,j} x_{ij}\, d_{ij} = \min .$$

This is discussed in the language of the personnel field [11], because that is where the model was first applied. It is now also being applied to production-scheduling problems in industry. Assume a supply of workers, say 2000 men monthly in the U. S. Air Force, to be assigned to jobs. The number of job classifications may be less than the number of men. It is also assumed known how many men are needed in each job classification and that there is a skill rating for each man under each job classification. The jobs will not here be divided into job classifications; the 2000 jobs will be considered all to be different. Formula (10) is written on this basis, with $m = n$ in our example and with d_{ij} measuring the skill rating of man i on job j. The "economic" condition now specifies that the men shall be assigned so as to maximize the sum of their individual skill ratings. In other words, now men are distributed to jobs efficiently, whereas, before, tankers were distributed to ports economically. All the previous remarks about the Distribution Problem apply to the Assignment Problem, as a special case.

Turn next to a Scheduling Problem, written as:

$$\sum_{i=1}^{m} x_{ij} = \sum_{j=1}^{m} x_{ij} = 1 , \quad x_{ii} = 0 , \quad \sum_{i,\,j} x_{ij}\, d_{ij} = \min ,$$

$$(11) \qquad x_{i_1 i_2} + x_{i_2 i_3} + \ldots + x_{i_{r-1} i_r} + x_{i_r i_1} \leq r - 1 ,$$

$$\text{for } r = 2, 3, \ldots, \left[\frac{m}{2}\right].$$

This is a much more difficult problem than (8) if we require integral solutions for x_{ij}. Although (11) is easily rewritten in the manner of (4), and is therefore a linear-programming problem, it turns out that the usual methods for solving linear-programming problems are of little or no use when integral solutions are required.* Fortunately, in both the Product Distribution Problem and the Assignment Problem there is an optimal solution in integers. Let us discuss this Scheduling Problem in another way before turning to the production-scheduling example.

This Scheduling Problem has been called the "traveling salesman" problem. For example, Formula (11) can be interpreted to represent the problem of going by air from Washington, D. C., to all 48 state capitols and then back to Washington, with minimum total mileage. The Scheduling Problem is to determine the order of visits so that the travel mileage is a minimum. At present, this problem is quite unsolved. For a dozen or so cities one can do it by trial and error, and for two dozen or so cities one can handle it nicely with modern computers. With 50 to 100 cities it is quite difficult, even with modern computers, and solution for 200 or more cities would probably be too difficult with present equipment and know-how.

*R. E. Gomory, in a paper presented during April 1958 at the New York City meeting of the American Mathematical Society, has given general methods for finding integral solutions of linear programs, and these methods may help on this problem.

To help understand the general Scheduling Problem, as represented by (11), consider a few simple examples in the geometric plane. If there are only two or three points to be reached on the tour, then there is only one possible path, since in this case the distance is the same in either direction between each pair of points. With four points, no three of which are collinear, it is easy to see that the shortest tour is described by the quadrilateral having the four points as vertices; this illustrates the general theorem, and one easily proved, that the optimal tour never crosses itself in this planar case, whatever the number of points. Even for a dozen points the number of possible paths is already so large that trial-and-error methods become quite awkward, even when advantage is taken of the theorem that the shortest path will not cross itself; and this one theorem, easily generalized to the case represented by (11), is the only one of any strength that has been proved.

We shall not present computational methods for solving (11), but it may be helpful to show how the technique used for solving (10) can be used in seeking a solution for (11). First of all, the condition $x_{ii} = 0$ can easily be dispensed with by increasing the values of d_{ii} sufficiently. Now, if the final condition of (11) is ignored, then what remains is a subproblem of form (10). It is a simple matter to check whether or not a solution obtained for this subproblem satisfies the final condition in (11); if it does then the calculation is finished, if it does not then the next step is to solve (11), including only those inequalities that were not satisfied by the solution of the initial subproblem, by using the general methods applicable to problems in form (4) for this purpose [6]. A continuation of this procedure will eventually lead to a solution of (11), and it may be further refined and accelerated by techniques not discussed here.* Our main purpose here has been to show the close connection between the Distribution Problem and the Scheduling Problem, while at the same time indicating the nature of the essential difference between them from the computational standpoint.

Consider the relevance of some of these ideas for a production-scheduling problem. We are interested in using a single machine tool in a sequence of specified operations, each requiring a different setup. It is known how long it takes to change from each setup to another, say d_{ij} minutes to change the setup from that for a job of "type i" to that for a job of "type j"; then d_{ij} is a measure of the "cost" of down time. It is assumed that several orders are on hand, each calling for certain specific machine operations. In this special case, it is also assumed that the operations may be done in any sequence and that the machine is returned to the starting setup upon completion of the orders, although this latter, very restrictive assumption would not often be a realistic one. The mathematical model is now complete for this kind of machine planning operation, and it is the same as in Formula (11).

This is the optimal tour problem: to move between setups on the machine in an order that requires minimum production time. These problems are quite realistic. A few comparisons of actual setup times in one plant with the setup times that would have resulted by applying the optimal tour solution showed a worthwhile improvement when the optimal solution was used. The rule used by the foreman in that plant is interesting. He considers all the jobs he has to do to fill present orders and then sets up for the one he can get to most quickly. This

*Dantzig, Fulkerson, and Johnson [7] have applied this general type of computational techniques in the successful analysis of a 49-city problem by using actual airline distances. Their methods have been applied successfully to several other numerical cases and seem to be reasonably effective. Other authors have published methods that sometimes work and that at least provide good approximations to the true solution; one of these by M. F. Dacey, but not yet published, has worked remarkably well on numerical examples.

sounds like a sensible rule. It is easy to show that there are better general rules, but there is as yet no practical solution to the problem: What is the best rule? This is still an open mathematical problem.

The several examples of linear programming theory, and their applications, have been discussed here in order to illustrate the three following points:

(1) If we are to use a mathematical model in solving a practical problem—a logistics operations problem—then it can often be a simple model, such as a set of linear equations, or linear inequalities, and one that is pretty well understood. It may still apply to a wide range of actual operating cases; and we have seen three or four examples.

(2) The model is always limited. It will never include all the factors. The models we have been discussing, in general, have not taken account of uncertainties in operations or of various other particular features.

(3) Even if we have a faithful model, the difficulties of wringing useful information from it may be very great. We saw this in the case requiring optimal scheduling of a machine tool. Furthermore, both the theoretical and the practical difficulties may vary widely between models having the same number of variables.

The reader may recognize this broad class of problems as essentially those of budget planning, which is a general way of viewing mathematical programming. If one has a prescribed set of things he wants to do, a fixed set of resources with which to do them, and can predict profits somehow in terms of the chosen amounts of activities, then one has a typical budget-planning problem. The linear-programming model is fundamentally one for use in budget planning: how to get maximum returns by selecting one from among several possible allocations of resources supporting a prescribed class of activities.

The product-distribution model, illustrated by the tanker fleet operation, has been applied with considerable success by several industrial organizations. The U. S. Armed Forces have been using the Personnel Assignment model. The Production Scheduling model is typical of several important applications in industry. For example, Melvin Salveson [25] has reported quite successful applications of this general kind of mathematical technique to production scheduling in the General Electric Company. We have seen that it is sometimes reasonable to expect that real savings can be made, even in highly developed production processes, as we realize how complex a task it is even to schedule a single machine tool optimally among 50 jobs.

Two rather different types of logistics operations problems that are currently receiving serious mathematical attention are discussed next. They have the same general virtues and faults as the mathematical programming examples already discussed, except that they give attention explicitly to uncertainty and its effects.

Formula (12) is a rather typical example of a simple Inventory Control model [8].

$$V(x, y) = cy + A[1 - F(y)] + \begin{cases} 0 & \text{if } y = x \\ K & \text{if } y > x, \end{cases} \qquad 0 \le x \le y < \infty; \tag{12}$$

where x = initial stock,

y = starting stock obtained by adding amount ordered to initial stock,

c = carrying cost per unit of goods,

A = the loss involved if impossible to meet demand for goods,

K = the ordering cost,

$V(x, y)$ = expected loss, and

$F(y)$ = distribution of demand.

The mathematical problem is to determine numerical values for the managerial variables x and y, in terms of the assumed known cost, loss, and demand quantities that will minimize the expected loss $V(x, y)$. It is by no means easy, in a real case, to determine the values of the constants c, A, K and the function $F(y)$. On the other hand, the manager must actually be making some such assumptions as background for his own intuitive calculations. Thus, in some cases, the over-all managerial-decision rule may be a better one if it is calculated mathematically in terms of careful explicit estimates of the determining constants [2,23,32].

The Preventive Maintenance problem is very similar to the Inventory Control problem, if we redefine the meaning of quantities in (12) somewhat as in Formula (13).

$$V(x, y) = C(y) + A[1 - F(y)] + \begin{cases} 0 & \text{if } y = x \\ K & \text{if } y > x, \end{cases} \tag{13}$$

where x = remaining service life of component,

y = starting service life of component,

$C(y)$ = cost of component with service life y,

A = the loss involved if component fails,

K = the component installation cost,

$V(x, y)$ = expected loss, and

$F(y)$ = mortality distribution.

The concept of number of items in stock is replaced directly by the concept of the remaining units of service time, and the concept of uncertain consumer demand is replaced by the concept of uncertain service use.

In the field of automation, the problems of preventive maintenance are becoming increasingly important. When one "automates" a plant, like the Ford Stamping Plant at Buffalo, there is considerable redesign needed for the dies. As automation leads to more uniform quality of product, there is much less need for the usual inspection and control steps along the production line. One large source of error has been reduced, namely, that due to human frailty. Tools are consequently redesigned to withstand harder use, but of a more uniform kind, and preventive maintenance schedules must reflect accurately both the changes in tool life and the greater importance of preventing the kinds of breakdowns previously forestalled and corrected by human operators. In the fully automatic plant, there would also have to be fully automatic maintenance processes!

Formulas (14) and (15) are examples of technological functions playing a role in the Preventive Maintenance model, very similar to that of F(y) in the Inventory Control model.

$$f(x)\,dx = \frac{\lambda^{\mu}}{\Gamma(\mu)}\, x^{\mu-1}\, e^{-\lambda x}\, dx\,, \quad (\mu > 0\,,\ \lambda > 0)\,. \tag{14}$$

$$df = -e^{\alpha}\, T^{\gamma-1}\, e^{-\alpha T}\,(\alpha T - \gamma)\,dT\,, \quad T = e^{-\beta t}\,, \tag{15}$$

where $\beta(\gamma - \alpha)$ = probability of failure due to impact on unweakened portion, and

$\beta\gamma$ = probability of failure due to impact on weakened portion.

Formula (14) is a mortality distribution that has been found by various authors to fit closely the failure data for a wide variety of items. Formula (15) is a mortality distribution, very closely related to the familiar Makeham - Gompertz actuarial function, that results from a rather simple technological model for the failure process [5,17].

There are now many examples of the use of equipment mortality functions in connection with design problems relating to reliability of complex equipments and systems. One interesting case is that of a new jet engine for use in airplanes, where the problem is to develop a reliable mortality function with only limited service experience available for use in estimating the relevant technological parameters. In such a case, it is not possible to make good estimates without some good idea as to the detailed form of the mortality distribution; for, otherwise, it would be necessary to wait until all of at least one whole generation of the new engines had failed in service even to estimate the mean engine life. Consequently, in addition to their use for preventive maintenance planning, such mortality-distribution functions are useful in planning spare-parts programs, in estimating system reliability, and in planning various other logistics operations.

Our main point about the Preventive Maintenance model is this: in the future, and particularly in the course of automation, there will be a substantial increase in the activity [12,18] of accurately estimating the probable length of life of components for use in maintenance planning and other logistics operations. This work will require an extensive use of mathematical models, representing the technology of the product and its components, in terms of parameters that have clear technological meanings after the fashion of Formula (15). There may even be more opportunity to use models like those in (12) to (15) in connection with problems of equipment design and maintenance than there would be with business applications in stock control. Practical success with such models requires that reliable cost, loss, and demand quantities be available, and the equipment technology often should be more readily available than are the economic data.

The Inventory Control model, as applied to Preventive Maintenance, emphasizes again our central point regarding mathematical methods when they are used in operations research and logistics studies:

> The same mathematical model can be and often is used for solving many different types of practical problems. Each such application takes advantage of logical properties of the model already established mathematically, and experience with each new type of application of any such model expands the area of future use.

As for the mathematical programming examples, the inventory-control examples show the great usefulness of a parameterized model for solving management problems.

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* * *

REDISTRIBUTION OF TOTAL STOCK OVER SEVERAL USER LOCATIONS*

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INTRODUCTION

Suppose it is desired to redistribute stock among several user locations so that the dollar cost of the expected total shortages as of some future time, say, the end of the current period, plus transportation cost, is minimal. It is assumed that the latter is, for any pair of locations, proportionate to the amount moved between this pair. With given assumptions about the probability distributions of demand, various properties of the solution to this problem are established, which permit the use of a very simple iterative method of computation. This procedure minimizes the objective function stated above, with respect to the first of a set of decision variables for given values of the remaining variables in this set, then with respect to the second variable after the indicated adjustment to the first is made, then with respect to the third variable, and so on. The method continues to cycle through the set of variables until a specified criterion is met. This procedure can be used for minimizing any member of a certain class of convex functions defined on p-dimensional intervals. Therefore, the characterization of the procedure is stated in terms of this more general problem.

With less general assumptions about unit transportation costs between locations, much simpler methods of solving the redistribution problem are obtained. In particular, the cases of zero costs and costs independent of origin and destination are examined.

STATEMENT OF THE PROBLEM

At time zero, consider the expected number of units short of a stock item at the kth location as of some future time T:

$$\int_{S_k}^{\infty} (y - S_k)\, d P_k (y) , \tag{1}$$

where

$$S_k = S_k^0 - \sum_{l=1}^{K} x_{kl} + \sum_{l=1}^{K} x_{lk} , \tag{2}$$

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and where

$P_k(y)$ = the probability that accumulated demand to time T at location k is no greater than y.

S_k = the stock at location k after redistribution at time zero is effected.

S_k^0 = the stock at location k before redistribution is effected (i.e., the initial stock).

x_{kl} = shipment from stock at location k to location l.

K = the number of locations stocking the item.

It will be assumed that $P_k(y)$ has a continuous derivative which is positive when y is positive.

Transportation cost of the redistribution $\{x_{kl}\}$ is assumed to be

$$\sum_{k=1}^{K} \sum_{l=1}^{K} c_{kl} x_{kl}, \tag{3}$$

where c_{kl} = unit transportation cost from location k to l.

Finally, it will be assumed that the cost is less for direct shipments between locations than indirect via a third location, i.e.,

$$c_{ij} < c_{ik} + c_{kj}. \tag{4}$$

If p denotes the cost of a shortage, then the sum F of shortage and transportation costs over all locations is

$$F = p \sum_{k=1}^{K} \int_{S_k}^{\infty} (y - S_k)\, dP_k(y) + \sum_{k=1}^{K} \sum_{l=1}^{K} c_{kl} x_{kl}. \tag{5}$$

A redistribution of stock is sought which minimizes total costs. More specifically, the problem is to find $\{x_{kl}\}$, such that

(6a) $x_{kl} \geq 0 \quad l = 1, \ldots, K; \quad k = 1, \ldots, K.$

(6b) the corresponding value of F is as small as possible subject to (6a).

At first, it may seem that the above is not an adequate statement of the problem, that restrictions are needed that relate to the fact that the maximal amount of stock which can be shipped from a location is fixed. It is obvious that assumption (4) implies that

$$x_{km} \cdot x_{lk} = 0 \quad k, l, m = 1, \ldots, K \tag{7}$$

is a property of the redistribution which satisfies (6a) and (6b), i.e., location k cannot both ship and receive stock. This suggests that the $\{x_{kl}\}$ should be additionally restricted by

$$\sum_{l=1}^{K} x_{kl} \leq S_k^0, \quad k = 1, \ldots, K. \tag{8}$$

An elementary deduction reveals that (8), like (7), is satisfied by the solution. This and other properties of the solution will be examined in the next section.

CHARACTERIZATION OF THE SOLUTION

To begin this section, a theorem is proved which permits a complete characterization of the redistribution satisfying (6a) and (6b).

THEOREM I: Let $g(x)$ be a real, convex, differentiable function of a point x in p-dimensional Euclidian space. Let $\left\{\begin{matrix} l_i \\ u_i \end{matrix}\right\}$ be a $\left\{\begin{matrix} \text{minimum} \\ \text{maximum} \end{matrix}\right\}$ of the ith coordinate x_i if x_i is bounded from $\left\{\begin{matrix} \text{below} \\ \text{above} \end{matrix}\right\}$. Let R be the set of all points so restricted. Then a necessary and sufficient condition that a point x^* minimize g over R is that

Property A: Either

$$\frac{\partial g(x^*)}{\partial x_i} = 0$$

or $$x_i^* = l_i \quad \text{and} \quad \frac{\partial g(x^*)}{\partial x_i} > 0$$

or $$x_i^* = u_i \quad \text{and} \quad \frac{\partial g(x^*)}{\partial x_i} < 0$$

is satisfied for $i = 1, 2, \ldots, p$.

PROOF: The necessity proof is trivial. To prove sufficiency, consider the following property of $g(x)$. For any given x^* and all x, for $0 \leq \alpha \leq 1$,

$$(1 - \alpha)\, g(x^*) + \alpha\, g(x) \geq g[(1 - \alpha)\, x^* + \alpha x]$$

or

$$g(x) - g(x^*) \geq \frac{g[x^* + \alpha (x - x^*)] - g(x^*)}{\alpha}. \tag{9}$$

Now as $\alpha \to 0$, the right-hand side of (9) approaches

$$\sum_{i=1}^{p} \frac{\partial g(x^*)}{\partial x_i}(x_i - x_i^*).$$

Hence, always,

$$g(x) \geq g(x^*) + \sum_{i=1}^{p} \frac{\partial g(x^*)}{\partial x_i}(x_i - x_i^*). \tag{10}$$

Suppose now x^* has Property A for all i. Then for every $x \in R$, $(x_i - x_i^*)\,\partial g(x^*)/\partial x_i \geq 0$, and, therefore, $g(x) \geq g(x^*)$, as was to be shown.

Note that the function F is a convex function of the variables $\{x_{kl}\}$. To prove this it is sufficient to observe that expression (1) is a convex function of the variable S_k, as is seen by differentiating twice with respect to the variable S_k. Since S_k is a linear transform of the variables $\{x_{kl}\}$, it follows that expression (1) is a convex function of $\{x_{kl}\}$. Finally, since F is the sum of terms, each of which is a convex function of $\{x_{kl}\}$, it follows that F is convex.†

Thus Theorem I applies to the redistribution problem. The solution $\{x_{kl}^*\}$ is completely characterized by the following:

Property A*: Either

$$\left.\frac{\partial F}{\partial x_{kl}}\right|_{\{x_{kl}^*\}} = 0$$

or

$$\left.\frac{\partial F}{\partial x_{kl}}\right|_{\{x_{kl}^*\}} > 0 \quad \text{and} \quad x_{kl}^* = 0,$$

where $\dfrac{\partial F}{\partial x_{kl}} = p\,[P_l(S_l) - P_k(S_k)] + c_{kl}$,

is satisfied for every pair (k, l).

Thus far, it would appear that the solution possibly requires use of Property A* to determine simultaneously values of as many as K^2 redistribution variables. However, a further observation reduces this number by more than half.

Suppose for some pair (k, l), $x_{kl}^* > 0$. Then from Property A* it follows that $p\,[P_l(S_l) - P_k(S_k)] + c_{kl} = 0$ and, hence, $P_l(S_l) < P_k(S_k)$. In view of (7), evidently $P_l(S_l) > P_l(S_l^0)$ and $P_k(S_k) < P_k(S_k^0)$. Therefore,

† But not strictly convex.

$$(11) \qquad x^*_{k1} > 0 \longrightarrow P_1\left(S^0_1\right) < P_k\left(S^0_k\right) \longrightarrow 1 - P_k\left(S^0_k\right) < 1 - P_1\left(S^0_1\right).$$

In other words, if location k ships to location 1, then the "initial" shortage probability at k must be smaller than at 1.

This result suggests a ranking of locations. Suppose each location is ranked in ascending order according to its initial shortage probability $1 - P_k(S^0_k)$, and suppose, without loss in generality, that the location subscript k is identical with the rank of the location by this criterion, i.e., "location 1" has the lowest initial shortage probability, "location 2" has the next lowest, while "location K" has the highest initial shortage probability. It follows that $x^*_{k1} = 0$ if $k \geq 1$ and therefore that there are at most $K(K-1)/2$ variables x_{k1} that can be positive. These variables (i.e., associated with the pairs of locations which may be involved in redistribution) can be identified by the ranking criterion.

It is also clear by Property A* that $x^*_{k1} > 0$ implies that the total shipments from k must be less than the initial stock at k, i.e.,

$$(12) \qquad x^*_{k1} > 0 \longrightarrow \sum_{1>k} x^*_{k1} \leq S^0_k.$$

For, if $\sum_{1>k} x^*_{k1} > S^0_k$, then $S_k < 0$, and therefore $P_k(S_k) = 0$; but for any 1 such that $x^*_{k1} > 0$, evidently $P_1(S_1) - P_k(S_k) = -c_{k1} < 0$. But then $P_1(S_1) < 0$, which is impossible. Hence $\sum_{1>k} x^*_{k1} \leq S^0_k$.

Property A* thus reveals a method for substantially reducing the number of variables to be considered in computation and also shows that a seemingly necessary a priori restriction, namely, inequation (8), turns out to be a property of the redistribution satisfying (6a) and (6b).

A COMPUTING PROCEDURE FOR CERTAIN CONVEX FUNCTIONS

Property A* also suggests an iterative procedure for obtaining the solution to the redistribution problem. This consists of satisfying this property, variable by variable, and continuing to cycle through the entire set of variables until a convergence criterion is met. This procedure, however, can be extended to a wide variety of minimization problems. Therefore, the following theorem of D'Esopo relating to this procedure is stated in general terms. The proof of the theorem may be found in Ref. [1].

THEOREM II. Assume:

(a) The region R and the function g are given as in Theorem I;

(b) In addition, for any two points x' and x'' in R such that x' and x'' differ only in values of one coordinate, let

$$g[\alpha x' + (1-\alpha)x''] < \alpha g(x') + (1-\alpha) g(x''), \quad 0 < \alpha < 1;$$

(c) $g(x) \to \infty$ as $x_i \to \begin{Bmatrix} -\infty \\ +\infty \end{Bmatrix}$ whenever x_i is unbounded from $\begin{Bmatrix} \text{below} \\ \text{above} \end{Bmatrix}$.

Consider the sequence $\{x^n\}$ defined by:

$$x_j^n = x_j^{n-1} + \delta_j \ , \ j = n \pmod p \ ,$$

(13)

$$x_i^n = x_i^{n-1} \ , \ i \neq j \ ,$$

where δ_j is chosen so that x_j^n satisfies Property A, and let M be the set of accumulation points of $\{x^n\}$.

Then:

(i) M is not empty,

(ii) Every $x \in M$ satisfies Property A for each coordinate, and hence minimizes g over R,

(iii) M has only one point if g is strictly convex,

(iv) $g(x^n) \downarrow \min_{x \in R} g(x)$,

(v) $\sum_{i=1}^{p} \min_{x \in R} \frac{\partial g(x^n)}{\partial x_i}(x_i - x_i^n) \to 0$ if R is bounded.

A special case of this theorem, where g is a positive definite quadratic form, was proved by Hildreth [2]. Attempts to lighten the rather heavy loading of hypotheses in the theorem have not been successful. They all appear necessary to insure that $g(x^n)$ converges to the minimum of g, even if x^n may not converge to a limit point.

To select a reasonable termination criterion for the procedure, it is desirable to state a bound on the reduction in g which is possible at any stage. The bound given below unfortunately requires that R be bounded.

From the proof of Theorem I, it can be observed that

$$g(x) \geq g(x^n) + \sum \frac{\partial g(x^n)}{\partial x_i}(x_i - x_i^n)$$

or

$$g(x^n) - g(x) \leq -\sum \frac{\partial g(x^n)}{\partial x_i}(x_i - x_i^n) \ .$$

If R is bounded, maximizing both sides of this inequality with respect to $x \in R$ yields

$$g(x^n) - \min_{x \in R} g(x) \leq - \sum \min_{x \in R} \frac{\partial g(x^n)}{\partial x_i} (x_i - x_i^n)$$

(14)

$$= - \sum \frac{\partial g(x^n)}{\partial x_i} (\bar{x}_i - x_i^n),$$

where

$$\bar{x}_i = \begin{cases} l_i & \text{if } \dfrac{\partial g(x^n)}{\partial x_i} \geq 0 \\ u_i & \text{if } \dfrac{\partial g(x^n)}{\partial x_i} < 0. \end{cases}$$

By Theorem II the right-hand side of (14) approaches zero and therefore may be used in deciding whether to stop computation.

APPLICATION TO THE REDISTRIBUTION PROBLEM

If F of expression (5) is twice differentiated with respect to x_{kl}, it is seen that the second hypothesis of Theorem II is satisfied for the function to be minimized in the redistribution problem. Furthermore, from property (12), R is bounded, since certainly the inequality $0 \leq x_{kl} \leq S_k^0$ must hold for all (k, l). Finally, in view of (11), it can be observed that the iterations need concern only the variables $\{x_{kl}\}$ for those pairs (k, l) where $P_k(S_k^0) > P_l(S_l^0)$. Let these variables be arranged in any given order for purposes of defining the iterations. Suppose that at the nth iteration a new value of x_{kl} is to be determined. If $S_j^{n-1} = S_j^0 + \sum_{m \neq k, l} x_{mj}^{n-1} - \sum_{m \neq k, l} x_{jm}^{n-1}$, for $j = k$ and $j = l$, then from Property A* either

$$P_k(S_k^{n-1}) - P_l(S_l^{n-1}) < \frac{c_{kl}}{p} \tag{15}$$

and $x_{kl}^n = 0$, or a nonnegative value of x_{kl}^n exists such that

$$P_k(S_k^{n-1} - x_{kl}^n) - P_l(S_l^{n-1} + x_{kl}^n) = \frac{c_{kl}}{p}. \tag{16}$$

If, for example, the P_j are assumed to be normal with mean μ_j and standard deviation σ_j, then solving Eq. (16) for x_{kl} is equivalent to solving

$$S_k^{n-1} - \mu_k - \bar{t}\sigma_k = \mu_l + \underline{t}\sigma_l - S_l^{n-1} \tag{17}$$

for $\bar{t}$, where $\underline{t}$ is the value of a function $f(\bar{t}, c_{kl}/p)$ defined by

$$\int_{\underline{t}}^{\bar{t}} \frac{e^{-(1/2)t^2}}{\sqrt{2\pi}}\,dt = \frac{c_{kl}}{p} \tag{18}$$

for $\bar{t} \geq \underline{t}$. The latter function is easily tabulated by graphical methods. The final solution satisfies

$$S_k - \mu_k - \bar{t}\sigma_k \leq 0 \leq S_l - \mu_l - \underline{t}\sigma_l \tag{19}$$

simultaneously for all pairs (k, l) such that $x_{kl} \geq 0$ and such that $(\bar{t}, \underline{t})$ satisfy (18) for each k, l. The sign = holds in (19) if $x_{kl} > 0$.

TWO SPECIAL CASES

In this section, the normality assumption for demand distributions will be retained. If, then, either unit transportation costs are zero, or if they are constant, independent of origin and destination, the solution to the redistribution problem is much more direct than in the general case.

If $c_{kl} = 0$ for all (k, l), then $\bar{t} = \underline{t} = t^*$ and $S_j = \mu_j + t^*\sigma_j$ for all j. Since $\Sigma S_j = \Sigma S_j^0$, it follows that

$$t^* = \frac{\Sigma S_j^0 - \Sigma \mu_j}{\Sigma \sigma_j}$$

Then the net shipments from the kth location are $S_k^0 - \mu_k - t^*\sigma_k$ if that quantity is positive, and the net receipts at the lth location are $\mu_l + t^*\sigma_l - S_l^0$ if that quantity is positive. The shipments between specific pairs of locations are indeterminate in this problem.

If $c_{kl} = c$ for all pairs (k, l), then from Property A* it is clear that

$$\max\,[P_k(S_k) - P_l(S_l)] = \max P_k(S_k) - \min P_l(S_l) \leq c/p.$$

Hence, if $\bar{P} = \max P_k(S_k)$, then $\bar{P} - c/p \leq P_k(S_k) \leq \bar{P} < 1$ for all k. If there is to be any redistribution at all, then the shippers consist of the set of locations

$$K_s = \left\{j: \quad P_j(S_j^0) > \bar{P}\right\}$$

$$= \left\{j: \quad \frac{S_j^0 - \mu_j}{\sigma_j} > \bar{t}\right\}$$

and the receivers

$$K_r = \left\{j: \quad P_j(S_j^0) < \bar{P} - c/p\right\} = \left\{j: \quad \frac{\mu_j - S_j^0}{\sigma_j} < \underline{t}\right\},$$

where $\underline{t} = f(\bar{t}, c/p)$. If redistribution does occur, then $\max P_k(S_k^0) - \min P_l(S_l^0) > c/p$. In this case, define

$$X_s = \sum_{j \in K_s} (S_j^0 - \mu_j - \bar{t}\sigma_j)$$

and

$$X_r = \sum_{j \in K_r} [\mu_j + f(\bar{t}, c/p)\sigma_j - S_j^0] .$$

It is easy to see that X_s strictly decreases from $\sum_{j=1}^{K} S_j^0$ to zero as $\bar{t}$ varies from $\min (S_j - \mu_j)/\sigma_j$ to $\max (S_j - \mu_j)/\sigma_j$. On the other hand, X_r increases from zero to a positive value. Hence, there is a unique $\bar{t}$ which satisfies $X_r = X_s$. Simple linear interpolation may be used to compute $\bar{t}$ as accurately as desired.

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* * *

LINEAR MODEL FOR EVALUATING COMPLEX SYSTEMS*

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ABSTRACT

The first phases of a long-range project for developing models for the evaluation of intelligence systems in terms of measurable outputs are reported. In making such evaluations, the number of correct identifications, number of false alarms, number of displacements, and number of complete misses must be measured, in each of several categories (such as infantry, artillery, armor, supply lines), in each of a number of sectors (front line, rear, far rear), and in each of several situations (such as attack or defense, good or poor mobility). The major results to date are (1) the development of experimental techniques, (2) the formulation of an analytical model, (3) the development of analysis techniques for evaluation of parameters in the model in terms of empirical results, (4) the preliminary evaluation of parameters (rank-order correlations of 0.8 and higher have been established between some of the analytic predictions and the empirical results), and (5) formulation of several concrete unsolved problems out of the over-all problem. Plans for continued study are discussed.

GENERAL CONSIDERATIONS

Introduction

In both war and peace military leaders are forced to make decisions regardless of whether or not they have all the information they would like on which to base their decision. Much current research, both mathematical and experimental, is devoted to the study of such decision problems under uncertainty. Some of this research concerns statistical and game theoretical techniques for arriving at decisions under conditions of uncertainty, and other research concentrates on techniques for reducing the amount of uncertainty. The work here reported falls primarily into this second category.

This study† is the beginning of a long-range attack on the problem of building a model for evaluating an intelligence system in terms of its total outputs as presented to the decision-making officer (at all levels). The parameters in the model would serve as measures of the relative importance of various types of information and so would be of aid in preliminary evaluation of proposed equipment for gathering information.

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The basic idea of the model reported on here is to achieve a good linear approximation for a nonlinear situation by splitting a large problem into many small pieces. The score given to an intelligence picture will be a linear combination of the scores each of its small pieces. The score assigned to a small piece must be computable in terms of the basic variables: hits, false alarms, and displacements. The model does not require that the score assigned to each small piece be linear in these basic variables, but it does seem desirable to keep this scoring as nearly linear as possible.

A central feature of the model is the use of judgments of experienced officers for determining the parameters in the scoring equations. One of the main contributions of the work thus far is the development of procedures for obtaining the necessary judgments.

One of the most difficult aspects of the problem of evaluating a surveillance system is the measurement of the accuracy and the importance of the information it provides.

A battlefield-surveillance system includes many sources of information about the enemy, such as aerial photographs, prisoner-of-war interviews, heat detectors, patrols, and radar. The output of each component takes the form of information (correct or incorrect) on such items as the identification of enemy units, their location, and their strength. The output of one component results in intelligence which will not in general be identical with that resulting from another component. For this and other reasons there arises the difficult problem of evaluating a component or even an entire system.

There are many criteria which should enter into the comparative evaluation of such systems. These may be classified under the headings of performance, military feasibility, technical feasibility and logistics. Many considerations under these classifications, such as speed, all weather capability, mobility, and manpower requirements, are objectively observable and not very difficult to measure. Much more difficult is the determination of how to combine judgments based on these many considerations to yield an over-all evaluation of a component or a system.

Scope for the Model

The model, which is formally presented below, was designed to be used in solving problems of the following type: given an enemy situation and an estimate of it, how may one determine (measure) the merit of the estimate. An estimate may (1) correctly identify and locate certain enemy units, (2) report some nonexistent units, (3) report certain actual units but give an incorrect location for them, and (4) fail altogether to detect certain units. These are called, respectively, hits, false alarms, displacements, and misses, and each is a basic variable in the model. They must be combined to obtain a measure of accuracy that enters into the final "value" of an intelligence estimate. Each is monotonically related to the value, but its relative contribution is not known (a priori). For instance, increasing the sensitivity of a detector might not only increase hits and decrease misses (which would contribute to value) but could also increase false alarms (which diminishes value). The value of the detector and its optimum sensitivity then depends in part on the relative importance of hits, false alarms, displacements, and misses.

In addition to constructing parameters which measure the relative importance of hits, false alarms, displacements, and misses for a given category of information, such as infantry units or artillery units, there is the problem of measuring the relative importance of information in the several categories. Such measures are needed to judge between two systems, one

of which is, for example, superior in detecting personnel and the other of which is superior in detecting motor vehicles.

Finally, there is another group of factors which must be taken into account in determining the value of an intelligence display. This group has to do with the character of the terrain and the depth of the sectors to which the display refers. It is reasonable to expect that the relative weights of hits, false alarms, displacements, and misses within a category of information, and the relative importance of the several categories of information, will vary from sector to sector. For example, an enemy airfield on the front line, where it is unusable, is relatively unimportant compared with an active light-artillery concentration in the same area, but the reverse is true fifty miles to the rear.

In summary, then, the major aspects of a surveillance system which will be considered here as contributing to the value of its intelligence display will be:

(1) The relative importance of hits, false alarms, displacements, and misses within a given category of information and within a given sector,
(2) The relative importance of the several categories of information within a given sector, and
(3) The effect of the nature of the sector upon aspects (1) and (2).

It is the purpose of our model to provide analytic expressions for these relative values. The basic parameters of the model are derived from subjective judgments made by military officers with considerable combat and staff experience. Consequently, a statistical procedure has been set up for converting the subjective judgments of high-level military personnel into a numerical scale which is used in determining these parameters. This procedure also contains built-in statistical checks on the feasibility and validity of the model.

THE MODEL

Sectors and Categories

The first step in building the model is to specify the various types of sectors and categories of information. It is clear that the class to which a sector belongs is determined in part by depth (e.g., distance from the front line), trafficability, and cover. However, interest in a given enemy sector also depends upon whether our mission is attack, holding, or withdrawal. In this first study only the attack mission is considered.

Additional distinctions might be introduced, but each new distinction greatly increases the labor of experimentally determining the parameters of the equations without affecting the theoretical aspect of the model. It is to be hoped, for purely practical reasons, that no great variety of situations need be considered.

The subdivision into sector types must, at least initially, be made on an a priori basis. When equations have been fitted to each sector type it may be observed that the parameters do not differ significantly among certain of them, in which case these sector types may be identified. On the other hand, a poor fit of the equation for some sector type may indicate the need for a further breakdown into subtypes. So the model is set up with a preliminary categorization into sector types with the expectation that modifications will be made as experience with the model dictates.

Initially, depth is divided into four intervals: 0-3000 yd (battalion and regiment), 3000-15,000 yd (division), 15,000-40,000 yd (corps), 40,000 yd - 200 mi (army and theater). Trafficability is divided into three degrees: (1) very good, as in open flat country with a road

network; (2) medium; and (3) very poor, as in marshy country. Similarly, cover is divided into three degrees: (1) heavily wooded, (2) some cover, and (3) open and unprotected. Thus, there are, potentially, 36 distinct types of enemy sectors. However, it is anticipated that some of these distinctions will be unimportant, and therefore some of the types can be combined. Sectors are designated by superscripts and the number of sectors is denoted by r. (Note that there may be many instances of sectors of a given type.)

The items about which information is obtained are broken down into categories. Subscripts are used to designate these categories and the number of categories is denoted by n. These categories include such items as artillery, tanks, infantry units, and airfields. Experience with the model will reveal whether some categories may be merged and others need to be divided further.

The Basic Equation

The general model is built around certain measurable integers which refer to information by category and sector. For a fixed sector j, t_i^j denotes the number of units of category i (i = 1, ... , n) in this sector, and the true situation vector in sector j is defined to be

$$T^j = (t_1^j, t_2^j, \ldots, t_n^j). \tag{1}$$

In a given intelligence display, c_i^j denotes the number of units of category i in the sector j which are correctly identified and located, and the correct identification vector in sector j is defined to be

$$C^j = (c_1^j, c_2^j, \ldots, c_i^j, \ldots, c_n^j). \tag{2}$$

In this same display there may be some indications of targets which in fact are not present. Such an error is called a false alarm, and a false alarm vector is defined by

$$E^j = (e_1^j, \ldots, e_i^j, \ldots, e_n^j), \tag{3}$$

where e_i^j is the number of false identifications of category i in sector j.

A displacement vector is defined by

$$S^j = (s_1^j, \ldots, s_i^j, \ldots, s_n^j), \tag{4}$$

where s_i^j is the number of units of category i in sector j which are correctly identified but incorrectly located. These four types of vectors are the variables in the model.

The number of misses is now given by the vector

$$M^j = (m_1^j, \ldots, m_i^j, \ldots, m_n^j), \tag{5}$$

where $m_i^j = t_i^j - c_i^j - s_i^j$.

Two further vectors serve as weighting factors. The first of these is

$$P^j = (p_1^j, \ldots, p_i^j, \ldots, p_n^j), \tag{6}$$

where p_i^j measures the relative importance of knowledge of a unit of category i in sector j. The second is

$$D^j = (d_1^j, \ldots, d_i^j, \ldots, d_n^j), \tag{7}$$

where d_i^j gives a category-wise evaluation of a display in terms of hits, false alarms, displacements, and misses. The equation for d_i^j,

$$d_i^j = c_i^j - f_i^j\, e_i^j + g_i^j\, s_i^j - h_i^j\, m_i^j, \tag{8}$$

was constructed after some preliminary investigations discussed below, in which the displacement term was not included. In (8) the coefficients 1, f_i^j, g_i^j, and h_i^j are parameters for the relative weights of hits, false alarms, displacements, and misses. The number d_i^j is the net score for a category of information in a given sector.

Equation (8) is normalized so as to make the coefficient of c_i^j be 1. The effect of this is to choose as the unit of measurement the value of a correct identification of one unit of the given category. This unit of measurement is of course arbitrary. For example, the unit of measurement for infantry might be in terms of platoons or companies or any other military unit. If for some displays companies were used as units in determining the d_i^j and for other platoons were used, it would be necessary to determine a conversion factor to make the d_i^j from one set of displays comparable with the d_i^j from the other. In the initial stages of constructing this linear model, the conversion factor may be chosen a priori on the basis of relative manpower of the respective units.

Valuation Indices

A series of indices may now be constructed, representing evaluations of the outputs of a surveillance system or its components:

$$p_i^j \cdot d_i^j \tag{9}$$

is the net importance score for a category of information in sector j;

$$z^j = P^j \cdot D^j = \sum_j p_i^j \cdot d_i^j \tag{10}$$

is the net importance score of all information in a given sector;

$$w^j = P^j \cdot T^j \tag{11}$$

is the worth of a sector;

$$v^j = \frac{z^j}{w^j} \tag{12}$$

is the score on information in a given sector; and

$$v = \frac{\sum_j z^j}{\sum_j w^j} = \frac{z}{w} \tag{13}$$

is the score for a theater.

The value or importance of information about a particular category of information in a particular sector will depend on the command echelon evaluating the information. Information about a platoon of enemy infantry is of great concern to a front-line commander and of less concern to the divisional commander. Conversely, the precise location of an enemy airfield 100 miles in the rear is of much greater value to higher-command echelons than to lower ones. The importance vector P^j represents the relative importance of the several categories of information within a given sector from the point of view of a command echelon which is concerned with the welfare of all its subordinate units.

However, it is reasonable that a particular surveillance device may yield information more valuable to one command echelon than to another, and it is therefore desirable to construct an index which represents the relative values of an intelligence display to different command echelons. The distinction lies primarily in the weighting of different sectors corresponding to their relative importance to different command echelons. The weighting vector is defined as

$$A(k) = \left(a^1(k), \ldots, a^j(k), \ldots, a^m(k)\right), \tag{14}$$

in which a command echelon is represented by k and the sector designation by the superscript j.

The worth of a total intelligence display to a particular command echelon, k, is expressed by

$$u(k) = \sum_j a^j(k) \cdot z^j. \tag{15}$$

Analogues of formulas (11), (12), and (13) would be needed in a complete development of this topic. In the present paper the treatment of echelon measures is limited to this brief introduction and no further use is made of Eqs. (14) and (15).

These equations define a linear model with the vectors T^j, C^j, E^j, S^j objectively observed and with the vectors P^j and D^j to be empirically determined. Equations (8) and (10) are the basic equations. The parameters and range of applicability of these equations must be estimated from empirical data.

EXPERIMENTAL AND MEASUREMENT PROBLEMS

A number of special problems arise in applying the abstract model presented above. In order to solve for the parameters of Eq. (8) it is first necessary to obtain numerical measures

of the d_1^j. There is, of course, a variety of direct and indirect methods by which this might be done. The most direct method is to present a true situation and an estimate of it to qualified and experienced military personnel and to ask for a numerical rating of the net score for that category of information in that sector. A highly indirect method would be to prepare detailed operational plans based on the estimate of the situation alone and then have these operational plans evaluated independently in the light of the true situation. The method of rank-ordering estimates of a true situation and the method of pair comparisons fall between these two extremes. The numerical-rating method requires the assumption of a stable origin and a constant unit of measurement common to all officers. On the other hand, it is administratively simple and seems reasonable to the officers. Ranking and paired comparisons require necessarily somewhat less strong assumptions but are administratively more difficult. For the purposes of the preliminary testing of the model reported here, the method of rank order was decided upon as a feasible compromise. In this method the overlay containing the estimate and the map of the true situation constitute the "object" to be judged, and a set of these is ranked in order from best to worst with respect to how well the estimate approximates the true situation. The worst display is given the number 0, the next worst 1, and so on. The number thus assigned to the h-th display by the p-th officer is designated by $b_1^j(h, p)$.

These rank orders are obtained from each of a number of officers and averaged over the sample of officers for each display. The measure of the value (designated $\tilde{b}_1^j(h)$) of display number h is taken as

$$\tilde{b}_1^j(h) = \frac{1}{(N-1)\,M} \sum_{p=1}^{M} b_1^j(h, p) \quad (h = 1, 2, \ldots, N), \tag{16}$$

in which N is the number of displays and M is the number of officers used as judges. The range of $\tilde{b}_1^j(h)$ is 0 to 1; a perfect estimate would be given the score 1.

This system is not completely satisfactory. For example, for M = 1 the difference between the values assigned to any two objects is some multiple of $1/N - 1$, regardless of how small the "psychological distance" between these objects may be. Obviously, for M small, Eq. (16) will not adequately represent the relative values of two objects. However, as M increases, it is reasonable to expect that the percentage of judges who rank one object ahead of another will be a function of the "psychological distance" between them. That is, if two objects, A and B, have a sufficiently small "psychological distance" between them, one could expect the number of judges ranking A ahead of B to approach M/2 as M becomes large. In this case,

$$\left| \tilde{b}(A) - \tilde{b}(B) \right|$$

becomes small.

Another defect with this method becomes evident if an estimate of the corresponding situation is perfect. Then, presumably, every judge will rank this "object," which we may call A, as best, so $\tilde{b}(A) = 1$. Now suppose B is another object which, ranked on an absolute scale, would rank as almost perfect. By the above method, since $\tilde{b}(A) = 1$, $\tilde{b}(B) \leq 1 - (1/N - 1)$, so that Eq. (16) might distort the "psychological distance" between objects A and B. However, this defect is easily remedied by designing the experiment so that no estimate is perfect.

In the present experiment, one estimate of an artillery situation was taken as perfect. In a redesigned experiment no perfect estimates will be included.

FORT BENNING STUDY

The Pretest

A pretest was run on eleven officers to try out materials, test the instructions, estimate time limits, and obtain some preliminary notions of how well the model might work. After some minor revisions, materials and instructions were prepared and arrangements were made with the infantry school at Fort Benning for a number of officers with combat experience in WW II and in the Korean War to act as judges.

Test Material

Two categories of information were chosen: infantry companies and artillery batteries ($i = 1$ or 2) within a single sector, $j = 1$. The situation was an attack mission on the part of our forces, the terrain had very good trafficability with moderate cover, and the sector was a regimental front with a depth of 3000 yd. The actual terrain chosen was in the neighborhood of Fort Bragg, North Carolina, and three separate instances of the terrain were used: terrain instance I was the Marston map, II was Silver Hill, and III was Millstone Lake.

A number of maps of each terrain were secured on which were displayed a true enemy situation showing various dispositions of infantry companies only, of artillery batteries only, or of both infantry companies and artillery batteries. Then an estimate of the true situation was printed on an overlay and paired with the true situation. The pair were always kept together to constitute a stimulus for judgment.

Subjects

The judgments of 19 officers at Fort Benning were secured. There were four lieutenant colonels, fourteen majors, and one captain. Their combat command experience, obtained from interview data, ranged from a minimum of 3 months up to 40 months with an average of 12.6 months. They were also asked about their staff experience, particularly in Intelligence (G-2) or Operations (G-3) sections. Seven officers had no experience in G-2 or G-3 sections and one had 40 months. The mean was 6 months. Most of their experience had been at the company and battalion level but, being field grade officers in the regular army, they were now oriented toward regimental levels. However, their lack of regimental command experience raises a question about whether or not their judgments should be exclusively relied upon for determining the parameters of the equations.

Instructions and Procedure

After the experience with the pretest, a detailed and uniform set of instructions and procedures were prepared. The officers were scheduled for the test two at a time at intervals of two hours at 8 and 10 a.m. and 1 and 3 p.m. There were two complete sets of test materials, and the entire procedure rarely took more than four hours for any one officer.

The two officers beginning the test procedure at a particular time were first given a briefing covering the instructions and describing the procedure. This briefing was read to them in a casual and informal manner with discussion of questions if they arose.

Agreement Between Judges

As there were 19 judges, the conformity of agreement among the judges was measured by computing the τ coefficient* between the rank order of the stimuli by each judge and the average of the rank orders by the other judges. This was done separately for infantry, artillery, and for the combined infantry-artillery displays. These coefficients are reported in Table 1.

TABLE 1

Agreement of Each Judge's Rank Order with the Average Rank Order of the Remaining Judges (Fort Benning)

Officer	Infantry	Artillery	Combined
1	0.874	0.708	0.648
2	0.874	0.621	0.688
3	0.842	0.731	0.672
4	0.818	0.628	0.731
5	0.814	0.723	0.771
6	0.802	0.632	0.632
7	0.794	0.668	0.561
8	0.791	0.605	0.731
9	0.787	0.727	0.664
10	0.779	0.648	0.573
11	0.771	0.771	0.565
12	0.763	0.644	0.522
13	0.763	0.684	0.676
14	0.743	0.755	0.557
15	0.715	0.644	0.526
16	0.708	0.692	0.755
17	0.692	0.652	0.375
18	0.688	0.478	0.549
19	0.636	0.415	0.372

*M. G. Kendall, Rank Correlation Methods, Charles Griffin and Co., Ltd., 1943, pp. 26 and 81. Kendall's τ coefficient is a measure of the agreement of two rank orders of a set of objects. τ has a range of

$$-1 \leq \tau \leq 1$$

where $\tau = 1$ indicates perfect correlation and $\tau = 0$ indicates zero correlation.

To see if the agreement of an officer with the average of the other judges was specific as to whether infantry or artillery units were being judged, the rank orders by the officers on infantry situations were correlated with their rank orders on artillery situations. The τ coefficient* was 0.105, not significant, indicating that the degree of agreement of an officer's rank order of the infantry stimuli with the consensus of the remaining officers is not related to his degree of agreement on artillery stimuli. The corresponding τ coefficients between infantry on the one hand and artillery on the other with combined infantry-artillery stimuli were 0.462 and 0.129, respectively.

The W coefficient* is a measure of the agreement among several judges in rank ordering a set of objects and has a range of

$$0 \leq W \leq 1$$

where W = 1 indicates perfect agreement and W = 0 indicates no agreement.

TABLE 2
Community of Agreement in Rankings (Fort Benning)

Unit	W	ρ_{av}
Infantry	0.85	0.84
Artillery	0.71	0.69
Infantry-Artillery	0.65	0.63

The W coefficient for all 19 judges together was computed on infantry, artillery, and infantry-artillery combined. The results, presented in Table 2, indicate a significant degree of agreement.

None of these results provided any secure basis for eliminating any of the officers from the sample; however, the considerably lower correlations of the last three officers (17, 18, and 19 in Table 1) suggests the possibility that eliminating the data from them would improve the validity of the results.

FORT SILL STUDY

Reason for Fort Sill Study

In the analysis of the data obtained at Fort Benning, it was observed that the agreement among the officers' judgments on situations involving infantry alone was very good, while the agreement on situations involving artillery alone was poor (Tables 1 and 2). Since Fort Benning is an infantry school, it seemed possible that a bias was reflected in the judgments of its officers. Accordingly, it was decided to run a separate experiment at the Artillery School at Fort Sill and compare the results with those from Fort Benning. The same maps and overlays were used as at Fort Benning and the same procedure was followed.

Subjects

The judgments of eighteen officers at Fort Sill were secured. The average rank and staff experience of the officers involved at Fort Sill was somewhat higher than that of the officers tested at Fort Benning. There were two colonels, fifteen lieutenant colonels, and one major in the group. The average combat experience was 8.5 months, the average staff experience was 3.5 months, and the average noncombat staff experience was 13 months. Much of the experience was at battalion and regimental levels.

*See footnote, p. 355.

Agreement Among Judges

As in the analysis of the data from Fort Benning, the agreement among judges was measured by computing τ coefficient between the rank order of the stimuli for each judge and the average rank order for the other judges. The results for infantry, artillery, and combined infantry-artillery are reported in Table 3. Also, the τ coefficient of the officers' agreement among themselves for infantry and the officers' agreement among themselves for artillery was computed. The value obtained was -0.196, indicating that the degree of agreement of an officers' rank order of the infantry stimuli with the average rank order is not related to the degree of his agreement on artillery stimuli. The corresponding τ coefficient between infantry and combined infantry-artillery stimuli was 0.51. The W coefficient for all eighteen judges together is presented in Table 4.

TABLE 3
Agreement of Each Judge's Rank Order with the Average Rank Orders by the Other Judges (Fort Sill)

Officer	Infantry	Artillery	Combined
1	0.917	0.751	0.743
2	0.874	0.490	0.755
3	0.866	0.470	0.688
4	0.862	0.696	0.743
5	0.842	0.731	0.775
6	0.842	0.723	0.739
7	0.818	0.743	0.791
8	0.806	0.826	0.692
9	0.806	0.383	0.688
10	0.791	0.621	0.751
11	0.787	0.700	0.628
12	0.767	0.688	0.660
13	0.751	0.652	0.688
14	0.743	0.751	0.739
15	0.723	0.771	0.676
16	0.719	0.692	0.680
17	0.644	0.731	0.581
18	0.623	0.735	0.601

Analysis of Judgment Criteria in Artillery Situations

Figures 1, 2, and 3 show that the two groups of officers agreed very well on the infantry situations and on the combined infantry-artillery situations, but on situations containing artillery alone, agreement was not very good. This tendency was also evident within each group.

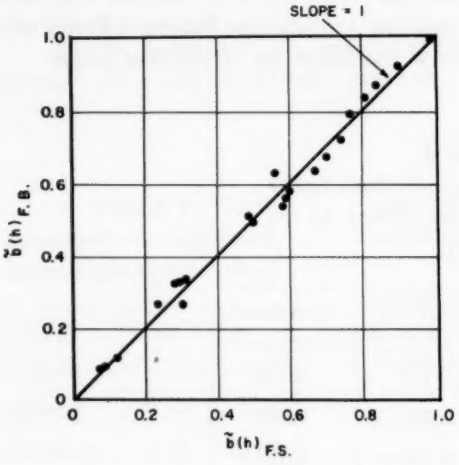

Figure 1 - Plot comparing average ranks of infantry situations from Fort Benning with those from Fort Sill

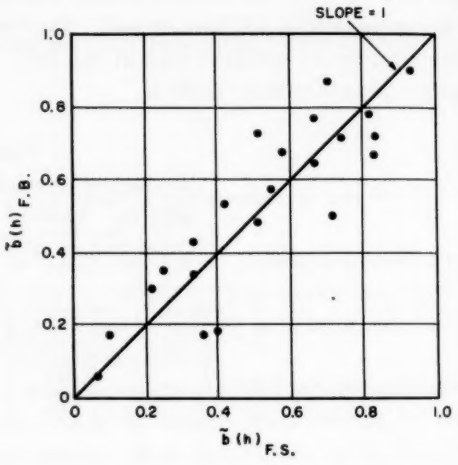

Figure 2 - Plot comparing average ranks of artillery situations from Fort Benning with those from Fort Sill

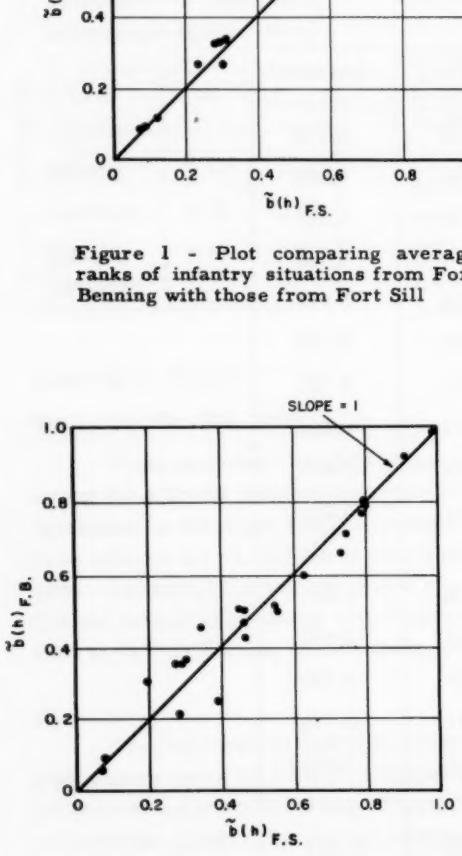

Figure 3 - Plot comparing average ranks of infantry-artillery situations from Fort Benning with those from Fort Sill

TABLE 4
Community of Agreement in Rankings (Fort Sill)

Unit	W	ρ_{av}
Infantry	0.87	0.86
Artillery	0.73	0.71
Infantry-Artillery	0.71	0.68

Some indications were obtained from the debriefing reports as to the reasons for the lack of agreement on artillery alone. The officers typically took one of two points of view—a concern for accuracy of reported strength or a concern for accuracy of reported location. The first point of view is essentially that of the commanding officer

concerned with making a decision on an appropriate course of action, the second that of an artillery officer concerned with counter-battery fire. Clearly these points of view would lead to different parameters for evaluating intelligence displays.

These results indicate the necessity for clearly specifying the echelon and situation for which the parameters are to be determined to insure that the evaluating officers will all take the same point of view.

CONCLUDING REMARKS

Suitable Model

As noted in the following section on "Some Unsolved Problems," in order to solve for the parameters f, g, and h, it was necessary to obtain a least-squares fit of a variant of Eq. (8) to the data. The $\bar{b}(h)$ values were taken to be empirical values for the intelligence estimates and were fitted to the equation

$$\hat{b}(h) = a_1 \frac{c}{\alpha} + a_2 \frac{e}{\alpha} + a_3 \frac{s}{\alpha}$$

by least squares. The term α is discussed below. Several of the forms discussed below were tested, and good fits to the raw data were obtained. Rank-order correlations of 0.8 and higher have been obtained between the $\bar{b}(h)$ and $\hat{b}(h)$ values for the intelligence estimates. These correlations are almost as high as is possible, in view of the degree of agreement among the judges.

Some Unsolved Problems

The method of collecting data gives individual rank orders of the stimuli (in this case intelligence displays). The average rank order of each stimulus was taken as a numerical measure of its value. The stimuli were selected so as to present a representative sample spread as uniformly as possible from very good to very poor.

One would expect that, if two stimuli are nearly equal in value, some judges would prefer one and some the other and that the amount of overlap could be used to judge the distance between them. Our use of the average rank order as an empirical score represents a primitive utilization of this idea. Further development of a measurement theory seems in order.

The basic scoring equation

$$d = c - fe + gs - hm \tag{17}$$

gives, for d, a theoretical range of

$$\min(-hm, -fe) \leqq d \leqq t. \tag{18}$$

It seems reasonable that the upper bound of the importance of a display should be the total number of units present. However, the linearity in e is subject to question for values of e much in excess of t. This suggests the desirability of replacing e in formula (17) by some function of e and of t which is bounded in e. For example, one proposal is

$$e\left(\frac{t}{t+e}\right)$$

or, more generally,

$$e\left(\frac{t}{t+ke}\right), \qquad 0 \leq k.$$

A still more general possibility is

(19) $$e \cdot F(t, e),$$

where F is a nonnegative function for which

(20) $$\lim_{t\to\infty} F(t, e) = 1$$

and

(21) $\lim_{e\to\infty} F(t, e)$ exists and is finite for all t.

The crux here is optimum selection of the function F.

Empirical evidence indicates that the judges do not make their preference choices on the basis of the d values but rather on $b = d/\alpha$ in which α is t or t + s, or t + e + s, or some similar quantity. Since α is a function of t, the number of units missed is now accounted for by the term c/α. In this case, the equation to be used is of the form

(22) $$b = \frac{c}{\alpha} - f\frac{e}{\alpha} + g\frac{s}{\alpha}.$$

Good fits to the data can be obtained with any of the above values of α. Further study, both theoretical and experimental, is necessary to discover the true nature of α. Further analysis of this problem is indicated.

Future Developments

In addition to work on the unsolved problems mentioned above, the following investigations are planned:

(A) There seem to be two major points of view represented in our artillery data. Officers in one group were interested primarily in pin-point location of enemy artillery so as to direct counterfire. Officers in another group were interested primarily in accurate knowledge of the number of opposing batteries. Clearly both points of view are important. However, the data are insufficient to give appropriate weights to these two factors. There are two possible approaches to this problem: (1) the use of higher ranking officers as judges or (2) a redesign of the experiment. The purpose in the first approach is to obtain judgments from officers who have been in command simultaneously of infantry troops and of artillery. The most probable course is to combine both approaches.

(B) A new set of displays should be designed to include at least three types of information and a larger front sector. Preparation of this new material will, of course, build on what has already been learned and will require continuing close touch with the military liaison. This material would be intended for judgments by officers of somewhat higher rank than were those in the earlier experiments. The value of the debriefing records from this experiment indicates the desirability of paying even more attention to this feature of military experience.

* * *

NOTES

PROBLEMS

It has been suggested that this journal might serve as a medium for publishing problems in the area of Logistics – with the idea that other persons might be interested in those problems and might submit comments. Readers are invited to submit brief statements on applied and theoretical problems in Logistics. Address letters to Managing Editor, Naval Research Logistics Quarterly, Office of Naval Research, Washington 25, D. C.

The following letter has been received:

19 September 1958

Dear Sir:

Three papers which concern a particular "assignment problem" have been published in the Quarterly. They are:

(1) "A Plan to Allocate and Procure Electronic Sets by the Use of Linear Programming Techniques and Analytical Methods of Assigning Values to Qualitative Factors" by Jack W. Smith in Vol. 3, No. 3, September 1956.

(2) "Procurement and Allocation of Naval Electronic Equipments" by George Suzuki in Vol. 4, No. 1, March 1957.

(3) "The Coefficients in an Allocation Problem" by R. J. Aumann and J. B. Kruskal in Vol. 5, No. 2, June 1958.

We wish to offer several comments on these papers.

The three papers are conservative in the sense that they do not purport to offer the only solution to the "electronics allocation problem." In their "Reservations and Conclusions" Aumann and Kruskal are particularly helpful in point out limitations and in suggesting that other techniques are conceivable and some of them might be preferable. As far as routine application of the procedure is concerned, the degree of optimism varies among the three papers. Smith appears to be the most and Suzuki the least optimistic.

Because of the authors' conservatism in this respect, it may be that they have considered the substance of the following comments but, if they have, the papers do not indicate it.

Our principal comment concerns the intent of the Material Improvement Plan (MIP) which is the basic source of information in the problem. Our questions here are:

(a) Is it the intent of the MIP that if it can be done, all Priority 1 requirements be fulfilled before Priority 2? Or,

(b) Is it the intent to maximize the total "effectiveness" of all the ships covered by the particular MIP with due consideration given to the specified "priorities?"

It seems clear that the papers adhere to the second policy in the sense that if Priority 1 required two "models" with a given set of characteristics and Priority 16 required two "models" with the same set of characteristics, an available model might be assigned to fill the first of Priority 16 rather than the second of Priority 1. This may be the intent of the MIP, but the assignment of priorities does not indicate it to be. The assignment of priorities in the MIP and in fact the derivation of the MIP is based on the "importance" of ship classes. "Importance" here refers to which classes of ships should first receive alterations or improvements.

A principal question used in the technique for assigning values is as follows [2, p. 6]:

"Suppose one equipment model A is available for assignment and there are two activity states to which the set can be assigned.

Activity state j (with importance mj) with equipment model B installed,

And activity state k (with importance mk) with equipment model C installed.

Where would you assign the available set A?"

Assuming that $m_j > m_k$, the answer is already provided by the MIP, namely that A would be assigned to activity state j. Now the approach suggested by the papers provides a different orientation. This is to consider the equipments already installed and if, for example (A - C)> (A - B) then A may be assigned to activity k. The relevance of including for consideration the equipments presently installed will be commented upon later. For the present, this difference in orientation will suffice.

Our second comment is that the terminology in the papers, particularly [1] and [3] is less clearly defined than it might be. The need for reasonably precise definitions may be illustrated by an outline of the problem as we see it.

The problem involves:

(a) Several "Priorities" as assigned in the MIP

(b) Each "Priority" defines one "system" (which Smith calls a "Requirement") e.g. Surface Search Radar, VHF Transmitter, Portable Transmitter.

Note: The hardware pertaining to one "system" is greatly different from that of another. A point not brought out by the authors.

(c) However, for each priority there are, in general, several "classes" of ships. A ship "Class" is defined by its "Type" (e.g., a CVA is a different "Type" from a CL which differs from a DDR) and a number. Within a "Type" there are several "Classes."

(d) There are several ships in each class.

(e) Within a given Priority – System – Class, the requirement may be for several identical models on the same ship.

(f) The individual "requirement" may be satisfied by several existing models.

With these many "severals" one readily becomes confused as to which "set" (in the mathematical sense) is being discussed.

This difficulty of understanding may be illustrated by the discussion in Section 6 of [3]. If the authors intend to "confront the board with an allocation problem" in which "b^i" means a requirement for a "surface search radar" and "b^j" means a "VHF Transmitter," with only one VHF Transmitter available, the real world problem is not being reproduced for b^i and b^j are incomparable. It is not likely that this is what the authors intended but for those who would like to employ the technique, a somewhat sharper exposition would be desirable.

Another comment on terminology is that the use by Aumann and Kruskal of such terms as "natural utility" and "physical utility" does not seem to be appropriate. For example, it is difficult to see how a counting of dollars to determine profit [3, p. 112] relates directly to an objective, physical, measurable utility.

Let us now return to the relevance of the technique and the "questions" proposed by the authors for the assignment of values. Earlier we noted a difference between what appears to be the intent of the MIP and that of the procedure suggested by the authors. It is important to pursue this further.

Some probable characteristics of the problem which have not been discussed by the authors are worthy of attention.

It seems that allocations under the MIP are concerned with alterations to vessels and not sheer replacements. If, for example, an equipment is not operating satisfactorily (i.e., not performing the function for which it was designed or performing it unreliably) and is to be replaced by another unit of the same type of equipment, this is not the concern of the MIP. The MIP relates to the allocation of equipments which are new as far as design and capabilities are concerned. Hence, to a significant degree "installed equipments" represent a "void" insofar as being able to perform some increased or different capability. Also alterations frequently result in a situation where a newly designed and manufactured equipment is to be a "net" addition to a vessel (i.e., none of the "installed equipments" are removed). Another point worth mentioning is that the design of new equipments to be installed aboard ship is not made in a vacuum. To some degree, the equipments are designed and built for a particular "system" and a particular "Type" and "Class" of ships. Finally there is the important point made earlier that equipments for one "system" are generally not applicable for another "system."

With these features in mind, we should like to offer the following summary comments:

(1) If we are dealing with two or more "systems," an equipment applicable to one "system" will generally not be applicable to another. This is the condition of incomparability.

For those relatively few cases where a particular equipment is applicable to two or more "systems," intransitivity could result. This is a possible condition not mentioned by the authors which would severely limit the use of the technique. For example, a situation such as the following could appear:

$$M_{1a} > M_{1b} > M_{2c} > M_{2b} > M_{1b} > M_{2a} > \text{-----}$$

(M_{ij} refers to equipment i on system j.)

If we are dealing with an equipment which may apply to several "ship classes" intransitivity may result. If the concern is an equipment which may be applied to several "systems" and several "classes" intransitivity will be more pronounced.

(2) If we are dealing with one "system" and two ship classes each with a different priority, we may have the following situation: let us make the following statements and assumptions for purposes of simplicity:

(a) There is only one ship in each class. The two classes are designated as a and b and it is assumed $a > b$.

(b) M_1 is an equipment installed on a and M_2 is installed on b.

(c) M_1 and M_2 apply to the same "system."

(d) M_1 has a capacity of 300 (e.g., 300 miles search radius) and M_2 has a capacity of 100.

(e) M_3 is a new equipment available with a capacity of 600 and can be installed either on a or b.

Under the MIP, M_3 would be installed on a even though M_1 already installed on a has a greater capacity than M_2. The reason for this is the higher priority given to a.

Under the technique suggested by the authors, M_3 may be installed on b since $M_3 - M_2 > M_3 - M_1$. The rationale here would be that the effectiveness of the fleet (the 2 ships) could be maximized.

For simplicity let us assume further that the capacities are additive. Then the total capacity of the installed equipments $(M_1 + M_2)$ is 400. If M_3 replaces M_1 the total capacity would be 700. If M_3 replaces M_2 the total capacity would be 900, hence the total effectiveness is maximized by this assignment. However, we question that this procedure is relevant.

If it has been decided that a specific class of ships should first receive equipments with a capacity of 600 and no equipments aboard these ships have this capacity, then independent of what is presently installed, this class of ships should be supplied with equipments with a capacity of 600. To do otherwise would seem to violate the intent of the plan. It may be that the intent of the plan should be changed but this is another matter and the authors of the papers (particularly Aumann and Kruskal) make a point of not attempting to violate naval judgment [3, p. 113].

If M_3 is installed on a the following may happen:

(a) M_3 may be a "net" addition to the vessel, i.e., M_1 may not be removed but left as a "spare" or "emergency" equipment.

(b) M_3 may replace M_1 and M_1 may replace M_2. This would result in a total effectiveness of 900, the same as replacing M_1 with M_3. This possibility would depend on a host of conditions. Among these are: whether or not M_1 and M_2 are considered obsolete and not worth doing anything with; the installation costs; availability of b (i.e., how soon b will be in an availability); the expected delivery rate for M_3, etc.

(c) M_3 may replace M_1 and M_1 may be placed in inventory as a spare for M_1's installed on other vessels; M_1 may also be "surplused" or cannibalized for parts.

One final remark worth mentioning is that the MIP seems to be a useful and unique type of document. It provides a basic plan for allocations and includes vital information (i.e., priorities) so that decisions can be made. If documents of this type were available for other areas in logistics (e.g., inventory management) some significant advances can be made.

In conclusion, it is our opinion that the papers in question are commendable first steps in the presentation and solution of the particular problem but, at least as judged from the papers themselves, there still remains the need for a sharper definition of the actual problem and further analysis before a reliable technique for its solution is evolved.

/sgd/ Ralph E. McShane
Henry Solomon

NEWS AND MEMORANDA

Readers are invited to submit to the Managing Editor items of general interest in the field of logistics

Thanks to the referees ....

The quality of the articles that appear in the NAVAL RESEARCH LOGISTICS QUARTERLY depends to a great degree upon the skill and interest of those who review the articles. We are grateful to the following persons who have acted as referees:

Arrow, K. J.
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Debrev, G.
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MARVIN I. ROSENBERG
Managing Editor

The following has been received:

THE RAND CORPORATION

Santa Monica, California

announces

A SYMPOSIUM ON MATHEMATICAL PROGRAMMING
(Linear Programming and Recent Extensions)

March 16-20, 1959

The Symposium will be under the general direction of George Dantzig. Among the topics to be discussed will be

Problem Formulation
Solution of Large Linear Programs
Nonlinear-programming Methods
Discrete Programming
Network Flow and Distribution Problems
Programming under Uncertainty

Papers are invited on these and related topics, both in the field of applications (March 16-17) and in the field of theory (March 18-20).

The Symposium will be unclassified and there will be no charge for attendance. Abstracts of about 100 words should be submitted before January 16, 1959.

Please send all abstracts, inquiries, and requests for registration forms to

DR. PHILIP WOLFE
THE RAND CORPORATION
1700 MAIN STREET
SANTA MONICA, CALIFORNIA

RECENT PUBLICATIONS

EDITOR'S NOTE: Listing of a publication in this section is for record and reference only and does not constitute an endorsement of point of view or advocacy of use.

REVIEW OF STUDIES IN THE MATHEMATICAL THEORY OF INVENTORY AND PRODUCTION. By K. J. Arrow, S. Karlin, and H. Scarf, Stanford University Press, Stanford, 1958, x + 340. $8.75.

This book is a collection of seventeen chapters written separately and jointly by the authors, with some contributions by M. J. Beckman, J. Gessford, and R. F. Muth. Although many of the chapters are closely related, they really constitute seventeen individual papers (three of them introductory), which can be read separately.

The general models considered are for the most part ones that are by now standard – for example, a majority of the articles deal with inventory problems of types which have appeared often in the literature in the last few years. The particular assumptions considered in the problems are, however, usually new, and the proofs are not merely the continual application of known techniques to new settings. Thus, even experts in this field should find the book interesting. The last four chapters, which are mainly concerned with limiting (in time) results, seemed like the most interesting part of the book mathematically.

The book invites comparison with Bellman's recent book, Dynamic Programming. The latter is designed as an elementary text more than the book under review, both in level of exposition and also in the inclusion of many problems for the reader. The book under review, on the other hand, goes much deeper into the economic background of the models, and the level of writing is usually that of a research paper rather than that of a text. Thus, the present book will make the more interesting reading for a mathematician-economist, while an engineering student will probably find Bellman's exposition and problems more to his liking.

The format of the book as a collection of papers has unfortunately led to the omission of an index, and to a bibliography which is only a supplement to those of Whitin and Gourary – Lewis – Neeland. Otherwise, it is a useful and interesting addition to the literature of the field, and the organization and distribution of topics is good despite the "collected" nature of the book.

(Reviewed by J. Kiefer)

LINEAR PROGRAMMING AND ASSOCIATED TECHNIQUES. By Vera Riley and Saul I. Gass, The Johns Hopkins Press, May 1958, 613 pages. $6.00.

This timely comprehensive bibliography on linear, nonlinear, and dynamic programming fills a definite current need. It should certainly be welcomed by workers in the fields of

mathematical programming, management sciences, operations research, logistics, and allied fields, as well as by others who are interested in becoming familiar with mathematical programming, and its extensive bibliography and applications. Most of the more then "1000 articles, books, monographs, documents, theses, conference proceedings, etc.," included is accompanied by a description of its contents.

The main emphasis is on linear programming, although the list of articles on dynamic programming appears to be fairly complete. Articles on the theory of games included in the book deal in large part with the relationship of zero-sum two person games to linear programming, although the important general references, and some mention of applications are included. The authors note that a complete bibliography on game theory in this book would not be appropriate.

The authors have striven valiantly, and with fair success, to anticipate the needs of users of the book, and have cataloged the literature topically, prefacing each section with introductory notes. These notes by themselves create a fairly good picture of the subject matter with which the bibliography is concerned. Quoting from the Foreward: "The format ..... encompasses the following elements: a brief description of the history and development of linear programming; an introductory presentation of the central problem (of linear programming) and a mathematical example; an assignment of references to subject categories; a definitive description of the compass of each category prefacing the individual sections; the inclusion of abstracts where they were already available; the annotation of the remaining references as needed; cross-filing of references to several of the more pertinent categories; the addition of an author index as an appendix."

The closing date for inclusion of references in the bibliography was arbitrarily set for June 1957, although the authors "succumbed to the temptation to slip in as many items of merit as the printer will allow."

The book is composed of four parts: Introduction, General Theory, Applications, and Non-linear and Dynamic Programming.

In the Foreward the authors bring out the fact that much of the usefulness of the book would have been forfeited by the many time-consuming revisions necessary to weed out all the small errors of omission and commission that one might expect in a work of this magnitude, and in an area where there is considerable confusion of terms. They thus decided that: "maximum usefulness would best be served by the earliest possible publication." Because of this, also, it is to be expected that individual readers may object to various points of presentation, emphasis, cataloging, etc.

Although grounds for these rather Picayune objections may exist, in the opinion of the reviewer their sum total would not offset the basic usefulness of the book. If we may anticipate continued rapid growth in the area of mathematical programming, with corresponding increase in publication, (one can hazard the guess that already a considerable number of articles have missed being mentioned in the bibliography under discussion) there will be the need for a further revision of the current edition, at which time many of these objections may be removed.

(Reviewed by C. E. Lemke)

THE HYPERCIRCLE IN MATHEMATICAL PHYSICS. By J. L. Synge, Cambridge University Press, 1957, pp 424. $13.50

This book will be of interest to the physicist, engineer, or mathematician who must solve partial differential equations subject to boundary conditions.

Using the concepts of function-space the author has developed a new technique in which the point of intersection of two orthorgonal linear subspaces is the solution of the problem. "The method of the hypercircle makes contact on one side with the theory of Hilbert spaces and on the other with equations in finite differences and the relaxation method of Sir Richard Southwell."

Neatly combining geometric intuition for guidance and algebraic methods for proof the author presents the hypercircle method in a function space with no metric in chapters 1 and 2, introduces a positive-definite metric so that the function space becomes a Hilbert space and discusses in detail the Dirichlet problem in a plane and the torsion problem in chapters 3 and 4 and various boundary value problems in chapter 5, introduces the function space with indefinite metric so that the geometry becomes analogous to that of Minkowski in chapter 6 and discusses vibration problems in this space in chapter 7.

The mathematics is precise and because only real functions are considered the Hilbert space is less difficult than the Hilbert space of quantum mechanics.

Exercises to test the reader are strategically placed throughout the book.

(Reviewed by J. S. Campbell)

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